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PHYSICS

NUMERICAL PROBLEMS & SOLUTIONS

PART 1 & 2

An initiative by

**ASSOCIATION OF PHYSICS TEACHERS
KOZHIKODE**

ASSOCIATION OF PHYSICS TEACHERS :KOZHIKODE

Numerical problems and solutions (Plus One)

chapter 1.Units and measurement

Question (1). A gas bubble from an explosion under water oscillates with a period T proportional to $P^a d^b E^c$, where P is the static pressure, d is the density of water and E is the total energy of explosion. Find the values of a, b, c ?

Solution

$$T = k P^a d^b E^c$$

$$[T] = [T^1]$$

$$[p] = [ML^{-1}T^{-2}]$$

$$[d] = [ML^{-3}]$$

$$[E] = [ML^2T^{-2}]$$

$$[T^1] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [ML^2T^{-2}]^c$$

$$[T^1] = [M^{a+b+c} L^{-a-3b+2c} T^{-2a-2c}]$$

Equating the powers on both sides

$$a+b+c = 0 \quad \text{.....(1)}$$

$$-a - 3b+2c = 0 \quad \text{.....(2)}$$

$$-2a - 2c = 1$$

$$-2(a+c) = 1$$

$$a+c = -\frac{1}{2}$$

$$-\frac{1}{2} + b = 0, \quad b = \frac{1}{2}$$

$$\text{eqn(1) + eqn(2)} \quad -2b + 3c = 0$$

$$c = \frac{1}{3}$$

$$\text{from } a + c = -\frac{1}{2}, \quad a = -\frac{5}{6}$$

solving the equations, $a = -\frac{5}{6}$ $b = \frac{1}{2}$ and $c = \frac{1}{3}$

Question (2). The velocity V (in cm/s) of a particle is given in terms of time t (in s) by the equation

$$v = at + \frac{b}{t+c}$$

Find the dimensions of a , b and c ?

Solution:

$$[v] = [at] = \left[\frac{b}{t+c}\right]$$

$$[at] = [LT^{-1}] \quad , \quad a = [LT^{-2}]$$

$$\left[\frac{b}{t+c}\right] = [LT^{-1}]$$

$$[c] = [T^1]$$

$$[b] = [L^1]$$

Question (3). Find the dimensions of a and b ?

$$E = \frac{b-x^2}{at^2}$$

where E is the energy, x is the distance and t is the time.

Solution:

$$[b-x^2] = [L^2]$$

$$[b] = [L^2]$$

$$\frac{L^2}{aT^2} = [ML^2T^{-2}]$$

$$[a] = [M^{-1}]$$

Question (4). Find the dimensions of a and b ?

$$F = a\sqrt{x} + bt^2$$

where F is the force, x is the distance and t is the time.

Solution:

$$[MLT^{-2}] = a L^{1/2} + b T^2$$

$$a L^{1/2} = [MLT^{-2}]$$

$$[a] = [ML^{1/2}T^{-2}]$$

$$b T^2 = [MLT^{-2}]$$

$$[b] = [MLT^{-4}]$$

Question (5). Check the correctness of the equation $\rho = \frac{3g}{4\pi rG}$

where ρ is the density, r is the radius, g is acceleration due to gravity and G is the gravitational constant.

Solution:

$$[ML^{-3}] = [LT^{-2}] / [L^1 M^{-1} L^3 T^{-2}]$$

$$= [ML^{-3}]$$

This equation is dimensionally correct.

Question (6) Time period of an oscillating drop of radius r , density ρ , k is a dimensionless constant

$$t = k \sqrt{\frac{\rho r^3}{s}}$$

check the correctness of the relation?

Solution:

$$[LHS] = [T]$$

$$[RHS] = \sqrt{\frac{[ML^{-3}][L^3]}{[ML^0T^{-2}]}] = [M^0L^0T^1]$$

Hence this equation is dimensionally correct.

Question (7). Find the number of significant figures?

- a) 0.007 m² b) 6.00320 J c) 2.640×10^{24} Kg d) 91.000m
e) 0.08240 N/m² f) 4.57×10^8 pa

solution:

- a)1 b)6 c)4 d)5 e)4 f)3

Question (8) The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively . Give the area and volume of the sheet to correct significant figures.

Solution:

area of metal sheet = $l \times b = 4.234 \times 1.005 = 4.25517 \text{ m}^2$

area of sheet to correct significant figures = 4.255 m^2

volume of metal sheet = $l \times b \times t = 4.234 \times 1.005 \times 2.01 \times 10^{-2} = 0.0855289 \text{ m}^3$

volume of sheet to correct significant figures = 0.0855 m^3

Question (9) A jeweller puts a diamond in a box weighing 1.2 Kg. The weight of the diamond is 5.42 g. Find the total weight of the box to correct significant figures.

Solution:

Weight of box a = 1.2 Kg.

Weight of diamond b = 0.00542 Kg.

Total weight = $a+b = 1.20542 \text{ Kg}$.

Total weight of the box to correct significant figures = 1.2 Kg

Question (10) Check the correctness of the relation by dimensional analysis

$$\lambda = \frac{h}{m \times v}$$

Solution:

$$[\lambda] = [L]$$

$$\left[\frac{h}{m \times v} \right] = \frac{[ML^2T^{-1}]}{[M][LT^{-1}]} = [L]$$

This equation is dimensionally correct.

Question (11) Check the correctness of the equation

$$v_c = \frac{R\eta}{D\rho}$$

where v_c is the critical velocity, R is dimensionless constant, D is diameter, ρ is the density, η is coefficient of viscosity ($[ML^{-1}T^{-1}]$).

Solution:

$$[v_c] = [LT^{-1}]$$

$$\left[\frac{R\eta}{D\rho}\right] = \frac{[ML^{-1}T^{-1}]}{[L][ML^{-3}]} = [LT^{-1}]$$

This equation is dimensionally correct.

Question (12) The rotational energy of a body is given by $E = \frac{1}{2}I\omega^2$, where ω is the angular velocity of the body. Use this equation to get dimensional formula for I ?

Solution:

$$I = \frac{2E}{\omega^2}$$

$$[I] = \frac{[ML^2T^{-2}]}{[T^{-1}]^2} = [ML^2]$$

Question (13) Check whether the given equation is dimensionally correct.

Capillary rise $h = \frac{r\rho g}{2S\cos\theta}$ where r is the radius, ρ is the density, g is acceleration due to gravity, S is the surface Tension.

Solution:

$$[h] = [L]$$

$$\left[\frac{r\rho g}{2S\cos\theta}\right] = \frac{[L][ML^{-3}][LT^{-2}]}{[MT^{-2}]} = [L^{-1}]$$

[LHS] not equal to [RHS]. This equation is not dimensionally correct.

Question (14) Obtain an expression for the excess pressure P inside a liquid drop using dimensional analysis

The factors depending are surface tension S and radius of liquid drop r ?

Solution:

$$P = K S^a r^b$$

$$[P] = [MT^{-2}]^a [L]^b$$

$$[M^1 L^{-1} T^{-2}] = [M^a L^b T^{-2a}]$$

$a = 1$, $b = -1$ and substituting in the first equation

$$P = \frac{KS}{r} \text{ where } K \text{ is constant}$$

Question (15) The escape velocity of a body depends upon acceleration due to gravity g on the planet and Radius of the planet R . Derive an equation for escape velocity using dimensional analysis.

Solution:

$$V_e = K g^a R^b$$

Taking dimensions on both sides

$$[LT^{-1}] = [LT^{-2}]^a [L]^b$$

$$[L^1 T^{-1}] = [L^a T^{-2a} L^b] = [L^{a+b} T^{-2a}]$$

Equating the powers on both sides, $a+b=1$

$$-2a = -1, a = 1/2$$

$$b = 1/2$$

$$V_e = K\sqrt{(gR)}$$

Question (16) State the number of significant figures in the following

(i) 0.0004 m² (ii) 4.32 x 10²⁴ Kg (iii) 0.2370 g/cm³

(iv) 6.320 J (v) 6.032 N (vi) 0.0006032

Solution:

(i) 1 (ii) 3 (iii) 4 (iv) 4 (v) 4 (vi) 4

Question (17) Round off to three significant figures

(i) 20.96 metre (ii) 0.0003125 Kg

Solution:

(i) 21.0 m (ii) 3.13 x 10⁻⁴ Kg

Question (18) A physical quantity is given by $h = \frac{F \times v^2}{L}$, F is the force, v is the velocity and L is the angular momentum. Find the dimension of h?

Solution:

$$[h] = [MLT^{-2}] \frac{[LT^{-1}]^2}{[ML^2T^{-1}]} = [M^0LT^{-3}]$$

Question (19) Check the correctness of the equation using principle of homogeneity of dimension?

$V = KA^2ut$ where K is a dimensionless constant, A is the area, u is the velocity and t is the time.

Solution:

$$[LHS] = [L^3]$$

$$[RHS] = [L^2]^2 [LT^{-1}] [T] = [L^5]$$

This equation is dimensionally wrong

Question (20) Write the dimensions of a/b in the relation :

$F = a\sqrt{x} + bt^2$ where F is the force, x is the distance and t is the time.

Solution: By principle of homogeneity of dimensions,

$$[a\sqrt{x}] = [F]$$

$$[a] = \left[\frac{F}{\sqrt{x}} \right] = \frac{[MLT^{-2}]}{[L^{\frac{1}{2}}]} = [ML^{\frac{1}{2}}T^{-2}]$$

$$[bt^2] = [F]$$

$$[b] = \left[\frac{F}{t^2} \right] = \frac{[MLT^{-2}]}{[MT^{-4}]} = [MLT^{-4}]$$

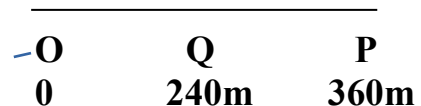
$$\left[\frac{a}{b} \right] = \frac{[ML^{\frac{1}{2}}T^{-2}]}{[MLT^{-4}]} = [M^0L^{\frac{-1}{2}}T^2]$$

Chapter 2: Motion in a straight line

Question (1) A car is moving along a straight line OP as shown in figure. It moves from O to P in 18s and returns from P to Q in 6s. What is the average velocity and average speed of the car in going from

- a) O to P b) O to P and back to Q

Solution: a) O to P



$$\text{Average speed} = \frac{\text{distance}}{\text{time interval}} = \frac{\text{path length}}{\text{time interval}} = \frac{360}{18} = 20 \text{ m/s}$$

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time interval}} = \frac{\text{final position} - \text{initial position}}{\text{final time} - \text{initial time}} = \frac{360}{18} = 20 \text{ m/s}$$

- b) O to P and back to Q

$$\text{Average speed} = \frac{480}{24} = 20 \text{ m/s}$$

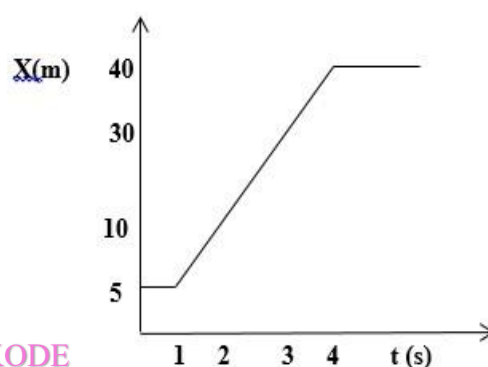
$$\text{Average velocity} = \frac{240}{24} = 10 \text{ m/s}$$

Question (2) A car travels from A to B at 60km/hr and returns to A at 90km/hr. What is the average velocity and average speed?

Solution: Average speed = $\frac{2v_1v_2}{v_1+v_2} = \frac{2 \times 60 \times 90}{60+90} = 72 \text{ km/hr} = 20 \text{ m/s}$

$$\text{Average velocity} = 0$$

Question (3) Position –time graph of a body is given



a) Estimate the velocity during the time interval $t=2\text{s}$ to $t=3\text{s}$

b) Displacement of an object is proportional to t^3 , show that its acceleration increases with time

Solution: a) velocity = slope of x - t graph in the time interval 2-3 s

$$v = \frac{30-10}{3-2} = 20 \text{ m/s}$$

b) $s \propto t^3$

$$v = \frac{ds}{dt} = \frac{dt^3}{dt} = 3t^2$$

$$a = \frac{dv}{dt} = \frac{d3t^2}{dt} = 6t$$

$$a \propto t$$

Question (4) Fig (1) shows displacement- time graph of runner A and fig (2) shows velocity-time graph of runner B

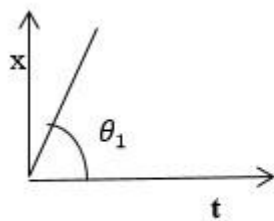


Fig (1)

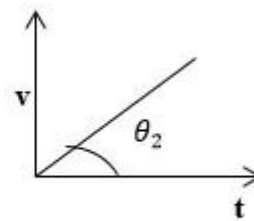


Fig (2)

a) Identify the type of motion the runner A has

b) Analysing the above two graph, find which runner will win the race. Why?

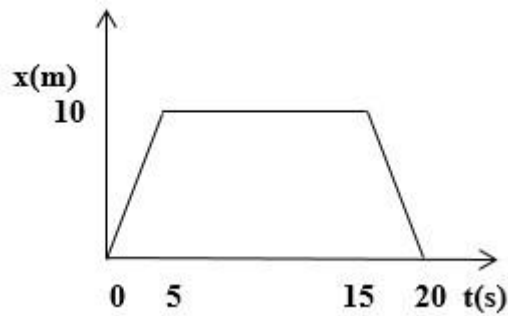
Solution: a) Uniform motion

b) Runner B will win the race. Because his velocity is increasing

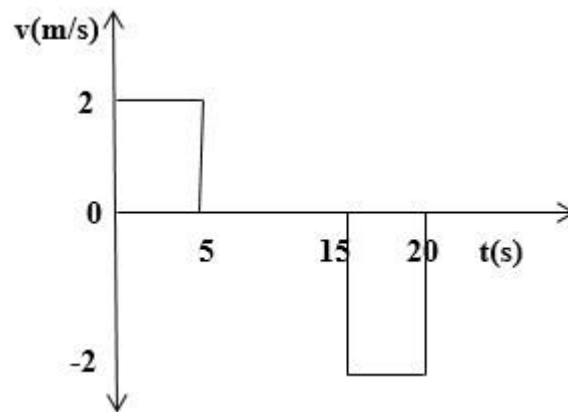
Question (5) The figure shows the position –time graph of a body moving along a straight line

a) Draw v - t graph

b) From the graph find the displacement in 20s

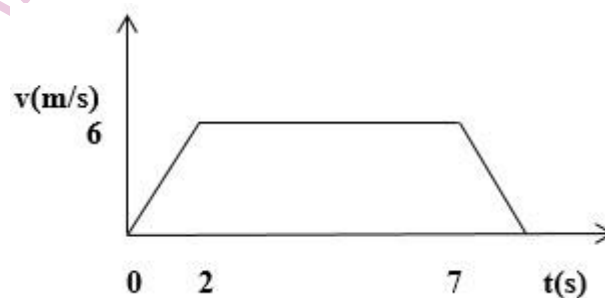


Solution a)



b) displacement = area under v-t graph
 $= (2 \times 5) + 0 + (-2 \times 5)$
 $= 0$

Question (6) v-t graph of a body is given below.



a) Which portion of the graph represents
 uniform acceleration
 uniform retardation
 and uniform motion

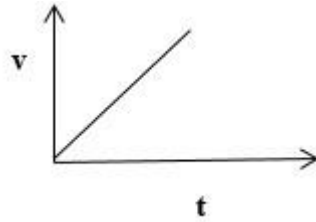
b) Find the displacement in the time 2s - 7s

Solution: a) OA, BC, AB

b) displacement = area under v-t graph

$$=(7-2) \times 6 = 30\text{m}$$

Question (7) Velocity -time graph of an object is given below



- a) What type of motion is indicated by the above graph
- b) Ratio of velocity to speed of an object is
 - i) one
 - ii) Greater than one
 - iii) Less than one
 - iv) Either less than one or equal to one

Solution:

- a) the graph indicates uniformly accelerated motion
- b) either less than one or equal to one

Question (8) If v is velocity and a is acceleration give an example of a physical situation for each of the following case

- a) $v \neq 0$, $a = 0$
- b) $v = 0$, $a \neq 0$
- c) $v > 0$, $a < 0$
- d) $v < 0$, $a > 0$

Solution:

- a) an object in uniform motion
- b) an object at maximum height
- c) an object moving in positive direction with decreasing velocity
- d) an object moving in negative direction with increasing velocity

Question (9) A player throws a ball vertically upwards with an initial speed of 29.4m/s .

- a) What is the direction of acceleration during the upward motion of the ball.
- b) What are the velocity and acceleration of the ball at the highest point? of its motion
- c) To what height does the ball rise
- d) How long does the ball take to reach the highest point?

Solution:

- a) Direction of acceleration during upward motion is down ward
- b) At the highest point of its motion

$$v=0$$

$$a=-g = -9.8\text{ms}^{-2}$$

To find height

$$v^2 - u^2 = 2as$$
$$0 - 29.4^2 = -2 \times 9.8 \times s$$
$$s = 44.1\text{m}$$

to find time

$$v = u + at$$
$$0 = 29.4 - 9.8t$$
$$t = 3\text{s}$$

Question (10) A coin dropped from the top of Leaning tower of Pisa reaches the bottom in 3.42s. What is the height of the tower

Solution:

$$s = ut + \frac{1}{2}at^2$$

$$h = 0 + (0.5 \times -9.8 \times 3.42 \times 3.42) = -57.31\text{m}$$

Question (11) A boy is playing with a ball on the terrace of a multi storey building of 25m height. Calculate the time the ball takes to reach the ground if

- a) he drops the ball
- b) he throws the ball vertically up with a speed of 20m/s
- c) he throws the ball vertically down with a speed of 20m/s

Solution:

a) free fall

$$s = ut + \frac{1}{2}at^2 \quad -25 = 0 - \frac{1}{2} \times 9.8 \times t^2 \quad t = 2.25\text{s}$$

b) Time taken to reach maximum height

$$v = u + at$$
$$0 = 20 - 9.8t$$
$$t = 2.04\text{s}$$

Distance covered in upward direction

$$v^2 - u^2 = 2as$$
$$0 - (20 \times 20) = 2 \times -9.8 \times s$$
$$s = 20.4\text{m}$$

Total distance from maximum height = $20.4 + 25 = 45.4\text{m}$

$$s = ut + \frac{1}{2}at^2$$

$$-45.4 = 0 + \frac{1}{2} \times -9.8 \times t^2$$

$$t = 3\text{s}$$

$$\text{total time} = 2.04 + 3 = 5.04\text{s}$$

c) $S = ut + \frac{1}{2}at^2$

$$-25 = -20t + 0.5 \times -10t^2$$

$$-25 = -20t - 5t^2$$

$$-5 = -4t - t^2$$

$$t^2 + 4t - 5 = 0$$

$$(t+5)(t-1) = 0$$

$$t = -5s \text{ or } t = 1s$$

$$t = 1s$$

Question (12) When brakes are applied on a moving vehicle, it stops after travelling a distance called stopping distance. If the initial speed is doubled keeping the retardation same, by how much will the stopping distance change

Solution:

initial stopping distance $S_1 = \frac{u^2}{2a}$

final stopping distance $S_2 = \frac{(2u)^2}{2a} = 4S_1$

change in stopping distance $= 3S_1$

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Chapter 3 -Motion in a Plane

Question (1): The resultant of two vectors P and Q is equal to R. On reversing the direction of Q, the resultant becomes S, Prove that $R^2 + S^2 = 2(P^2 + Q^2)$

Solution: Given, $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$ & $S = \sqrt{P^2 + Q^2 - 2PQ \cos \theta}$

Squaring both sides $\Rightarrow R^2 = P^2 + Q^2 + 2PQ \cos \theta \dots\dots\dots(1)$

$$S^2 = P^2 + Q^2 - 2PQ \cos \theta \dots\dots\dots(2)$$

Adding (1) & (2) $\Rightarrow R^2 + S^2 = 2(P^2 + Q^2)$

Question (2) : $\hat{i}$ and $\hat{j}$ are unit vectors along X and Y axis respectively. What is the magnitude and direction of the vector $\hat{i} + \hat{j}$?

Solution: $\hat{i}$ & $\hat{j}$ are unit vectors (ie, vectors having unit magnitude)
& also they are mutually perpendicular (ie, $\theta = 90^\circ$)

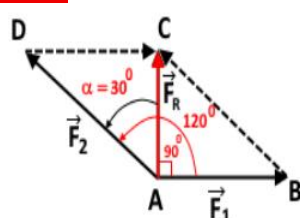
We have, resultant of two vectors p & Q, $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$
here, $P=1$, $Q=1$ & $\theta = 90^\circ$

$$\text{Then, } |\hat{i} + \hat{j}| = \sqrt{1^2 + 1^2 + 2 \times 1 \times 1 \times \cos 90^\circ} = \sqrt{2}$$

$$\text{direction, } \alpha = \tan^{-1} \left[\frac{Q \sin \theta}{p + Q \cos \theta} \right] = \tan^{-1} \frac{1 \times \sin 90^\circ}{1 + 1 \cos 90^\circ} = \tan^{-1}(1) = 45^\circ$$

Question (3) : The resultant of two vector acting at an angle of 120° is perpendicular to the smaller vector. If the larger vector is of magnitude 10 units, find the resultant and the smaller vector.

Solution:



$$\cos 30 = \frac{F_R}{10} \Rightarrow F_R = 10 \cos 30 = 8.66 \text{ Units}$$

$$\sin 30 = \frac{F_1}{10} \Rightarrow F_1 = 10 \sin 30 = 5 \text{ Units}$$

Question(4) : Find the angle between vectors A and B if $|A + B| = |A - B|$

Solution: Given, $\sqrt{A^2 + B^2 + 2AB \cos \theta} = \sqrt{A^2 + B^2 - 2AB \cos \theta}$

Squaring on both sides $\Rightarrow A^2 + B^2 + 2AB \cos \theta = A^2 + B^2 - 2AB \cos \theta$

$$4AB \cos \theta = 0 \quad \text{ie, } \cos \theta = 0, \quad \text{angle, } \theta = 90^\circ$$

Question(5) : Two equal forces have a resultant of $1.5 F$. Find the angle between the forces.

Solution: Given, $\sqrt{F^2 + F^2 + 2 \times F \times F \times \cos \theta} = 1.5 F$

$$\sqrt{2F^2 + 2F^2 \cos \theta} = 1.5 F$$

$$\sqrt{2F^2(1 + \cos \theta)} = 1.5 F$$

Squaring on both sides $\Rightarrow 2F^2(1 + \cos \theta) = 2.25 F^2$

$$2(1 + \cos \theta) = 2.25 \Rightarrow 1 + \cos \theta = 1.25$$

$$\Rightarrow \cos \theta = 0.25$$

$$\text{ie, angle } \theta = \cos^{-1}(0.25) = 72.52^\circ$$

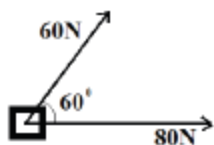
Question(6) : Two forces 5N and 10 N are acting at a point inclined at an angle of 120° Find the magnitude and direction of the resultant force.

Solution: Magnitude of resultant force, $F = \sqrt{5^2 + 10^2 + 2 \times 5 \times 10 \times \cos 120}$
 $= \sqrt{25 + 100 + 100 \cos(90 + 30)}$
 $= \sqrt{125 + 100 \times -\frac{\sqrt{3}}{2}}$
 $= \sqrt{125 - 50\sqrt{3}} = \sqrt{125 - 86.60} = \sqrt{38.4} = 6.197\text{N}$

direction, $\alpha = \tan^{-1} \left[\frac{Q \sin \theta}{p + Q \cos \theta} \right]$ where, $P=5\text{N}$, $Q=10\text{N}$ & $\theta = 120^\circ$

$\Rightarrow \alpha = \tan^{-1} \left[\frac{10 \sin(90+30)}{5+10 \cos(90+30)} \right] = \tan^{-1} \left[\frac{10 \sin(90+30)}{5+10 \cos(90+30)} \right] = \tan^{-1} \left[\frac{10 \sin(90+30)}{5+10 \cos(90+30)} \right]$

Question(7): Find the magnitude and direction of resultant force acting on the body in the given figure.

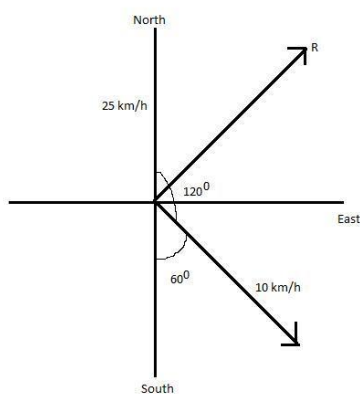


Solution:

$F = \sqrt{80^2 + 60^2 + 2 \times 80 \times 60 \times \cos 60} =$
 $= \sqrt{6400 + 3600 + 4800} = \sqrt{14800} = 121.655\text{N}$
 $\alpha = \tan^{-1} \left[\frac{60 \times \sin 60^\circ}{80 + 60 \times \cos 60^\circ} \right] = \tan^{-1} [0.47269] = 25.3^\circ$

Question(8): A motor boat is racing towards north at 25km/h and water current in that region is 10km/h in the direction 60° east of south. Find the resultant velocity of the boat.

Solution:



Resultant velocity, $v = \sqrt{v_1^2 + v_2^2 + 2v_1v_2 \cos \theta}$
 $=$
 $\sqrt{25^2 + 10^2 + 2 \times 25 \times 10 \times \cos 120}$
 $= 21.7\text{km/h}$

Question(9): Two forces $F_1 = 3\mathbf{i} + 4\mathbf{j}$ and $F_2 = 3\mathbf{j} + 4\mathbf{k}$ are acting simultaneously at a point. What is the magnitude of the resultant force?

Solution: $F_1 = 3\mathbf{i} + 4\mathbf{j} + 0\mathbf{k}$, $F_2 = 0\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$
 $F_1 + F_2 = 3\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$

Question (10): Find whether the given vectors $2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ and $4\mathbf{i}+6\mathbf{j}+8\mathbf{k}$ are parallel or not.

Solution: $A = 2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$
 $B = 4\mathbf{i}+6\mathbf{j}+8\mathbf{k}$
 $B = 2(2\mathbf{i}+3\mathbf{j}+4\mathbf{k})$
 $B = 2A$ ie, A is parallel to B

Question (11): Find the unit vector in the direction of the vector $\mathbf{i}+4\mathbf{j}-2\mathbf{k}$

Solution: Let, $\vec{A} = \mathbf{i}+4\mathbf{j}-2\mathbf{k}$, magnitude of $\vec{A}$, $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
 Where, $A_x = 1, A_y = 4$ & $A_z = -2$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\mathbf{i}+4\mathbf{j}-2\mathbf{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = \frac{\mathbf{i}+4\mathbf{j}-2\mathbf{k}}{\sqrt{1^2 + 4^2 + (-2)^2}}$$

 Unit vector in the direction of $\vec{A}$, ie, $\hat{A} = \frac{\mathbf{i}+4\mathbf{j}-2\mathbf{k}}{\sqrt{21}}$

Question(12): A particle starts from a point $A(1,2,-1)$ and reaches a point $B(3,2,2)$. Find the displacement vector $\vec{AB}$ and its magnitude.

Solution: Displacement = Position vector of B – Position vector of A
 $= 3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}} - (1\hat{\mathbf{i}}+2\hat{\mathbf{j}}-1\hat{\mathbf{k}})$
 $= 2\hat{\mathbf{i}}+3\hat{\mathbf{k}}$

Magnitude = $\sqrt{2^2 + 0^2 + 3^2} = \sqrt{13}$

Question(13): A boy can throw a ball to a maximum horizontal distance of 100m. With the same speed, how much high above the ground can he throw the same ball?

Solution: $R_{\max} = 2 \times H_{\max}$

$$H_{\max} = \frac{R_{\max}}{2} = \frac{100}{2} = 50\text{m}$$

Question(14): Ceiling of a hall is 25m high. What is the maximum horizontal distance that a ball thrown with a speed of 40m/s can go without hitting the ceiling of the hall?

Solution: Given, $H=25\text{m}$, $u=40\text{m/s}$, $g=9.8\text{m/s}^2$, $\theta = ?$, $R = ?$

$$H = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow 25 = \frac{40^2 \sin^2 \theta}{2 \times 9.8}$$

$$\sin \theta = \sqrt{\frac{25 \times 2 \times 9.8}{40^2}} \quad \theta = 33^\circ 36'$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{40^2 \times \sin 67^\circ 12'}{g} = 150.5\text{m}$$

Question(15): A ball thrown by a player reaches another player in 2s. What is the maximum height attained by the ball above the point of projection? (Take $g = 9.8 \text{ m/s}^2$)

Solution: Given, Time of Flight, $T = \frac{2u \sin \theta}{g} = 2\text{s}$ ie, $u \sin \theta = g$

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{(u \sin \theta)^2}{2g} = \frac{g^2}{2g} = \frac{g}{2} = \frac{9.8}{2} = 4.9\text{m}$$

Question(16): The horizontal range of a projectile thrown at an angle θ is R. Suggest another angle for which the range is the same.

Solution: The sine function has the property that $\sin(\theta) = \sin(180 - \theta)$.

$$\text{Since, } \sin(2\theta) = \sin(2(90 - \theta))$$

Therefore, if we replace θ with $(90 - \theta)$ in the range formula, the range remains the same

eg: If a projectile is thrown at an angle of 30 degrees, the same range can be achieved by throwing it at an angle of $(90 - 30) = 60$ degrees.

Question(17) : A bullet fired at an angle of 30° with the horizontal hits the ground 3km away. By adjusting its angle of projection, can one hope to hit the target 5km away? Assume the muzzle speed to be fixed.

Solution: we have, when, $\theta = 30^\circ$, $R=3000\text{m}$

$$\frac{u^2 \sin 2\theta}{g} = 3000$$

$$u^2 = \frac{3000 \times g}{\sin 2\theta} = \frac{3000 \times 9.8}{\sin(2 \times 30)} \Rightarrow u = 184.25\text{m}$$

$$R_{\max} = \frac{u^2}{g} = 3464.203\text{m} = 3.464\text{km} \text{ The bullet does not hit the target}$$

Question(18) : To obtain a horizontal range of 2000m, what is the least velocity of projection?

Solution: $R_{\max} = 2000\text{m}$ but, $R_{\max} = \frac{u^2}{g}$

$$u = \sqrt{R_{\max} \times g} = \sqrt{2000 \times 9.8} = 140 \text{ m/s}$$

Question(19) : The maximum Height attained by a shell is 8m and the horizontal range is 320m .find the Velocity & Angle of Projection. (Ans: 56m/s and 45°)

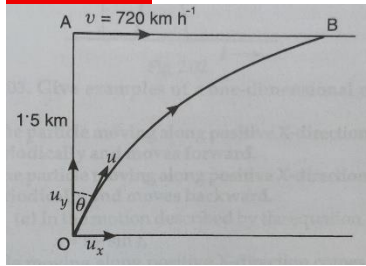
Solution: Given, $\frac{u^2 \sin^2 \theta}{2g} = 80\text{m} \dots\dots\dots(1)$

$$\frac{u^2 \sin 2\theta}{g} = 320\text{m} \dots\dots\dots(2)$$

$$(1)/(2) \Rightarrow \tan \theta = 1 \quad \text{ie, } \theta = 45^\circ \text{ substitute value of '}\theta\text{' in eqn.(2)}$$

$$\Rightarrow u = 56\text{m/s}$$

Question(20) : A fighter plane flying horizontally at an altitude of 1.5 km with a speed of 720 km/h passes directly over an anti –aircraft gun . At what angle from the vertical should the gun be fired , for the shell with muzzle speed 600m/s , to hit the plane ? At what minimum altitude should the pilot fly the plane to avoid being hit.

Solution:

Velocity of the plane = 200 m/s

Let the shell be fired at an angle θ with the vertical

Horizontal component of the velocity of the shell

$$= 600 \sin \theta$$

Vertical component of the velocity of the shell =

$$600 \cos \theta$$

For the shell to hit the plane, the horizontal distance covered by the shell and the plane must be in the same time 't'

$$600 \sin \theta = 200 t$$

$$\sin \theta = 0.3333 t \quad \therefore \theta = 19^\circ 28'$$

To avoid hitting, the plane should fly above the maximum height that can be reached by the shell.

Initial vertical velocity, $u = 600 \cos 19^\circ 28'$

Final vertical velocity, $v = 0$, $a = -9.8 \text{ m/s}^2$, $s = ?$

Consider the equation of motion, $v^2 - u^2 = 2as \Rightarrow s = 16.32 \text{ km}$

The plane should fly at an altitude above 16.3 km to avoid being hit.

Question(21): A aircraft executes a horizontal loop of radius 1 km with a steady speed of 900 km/h. Compare its centripetal acceleration with acceleration due to gravity.

Solution: $r = 1000 \text{ m}$, $v = 250 \text{ m/s}$, $a = ?$

Centripetal acceleration $a = \frac{v^2}{r} = 62.5 \text{ m/s}^2$

Acceleration due to gravity, $g = 9.8 \text{ m/s}^2$ $\omega = \frac{2\pi}{T} = \frac{2\pi}{60 \times 60} \text{ rad/sec}$

$$\therefore \frac{a}{g} = 6.4$$

Question(22): Calculate the angular speed of (i). Second hand, (ii). Minute hand and (iii). Hour hand of a watch.

Solution: (i). Second hand : $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} \text{ rad/sec}$

(ii). Minute hand : $\omega = \frac{2\pi}{T} = \frac{2\pi}{60 \times 60} \text{ rad/sec}$

(iii) Hour hand : $\omega = \frac{2\pi}{T} = \frac{2\pi}{12 \times 60 \times 60} \text{ rad/sec}$

Question(23): A ball, trapped in a circular path of radius 10 cm moves steadily and completes 10 revolutions in 100 seconds. What is the angular velocity of its motion?

Solution:

$r = 0.1 \text{ m}$, frequency, $\nu = \frac{n}{t} = \frac{10}{100} = 0.1 \text{ Hz}$,

Angular frequency, $\omega = 2\pi\nu = 0.628 \text{ rad/s}$

Linear velocity, $v = r\omega = 6.28 \times 10^{-2} \text{ m/s}$

Question (24): A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds. what is its acceleration?

Solution: $r = 0.8\text{m}$ $n = \frac{14}{25}$ $a = ?$

Centripetal acceleration,

$a = r\omega^2 = r(2\pi n)^2 = 4\pi^2 n^2 r = 9.894\text{m/s}^2$, direction is towards the centre of circle.

Chater 4 : LAWS OF MOTION

Question (1). A cricket ball of mass 0.2kg moving with a velocity of 20m/s is brought to rest in 0.4 second. Find the average force.

Solution:

mass of the cricket ball, $m = 0.2\text{kg}$

Initial velocity of the ball, $v_o = 20\text{m/s}$

Final velocity of the ball, $v = 0$

Acceleration of the ball $= \frac{v-v_o}{t} = \frac{0-20}{0.4} = 50\text{m/s}^2$

If F is the average force to stop the ball, $F = ma$

$F = 0.2 \times 50 = 10\text{ N}$

Question (2). Give the magnitude and direction of the net force acting on a stone of mass 0.1 kg,

(a) just after it is dropped from the window of a stationary train

(b) just after it is dropped from a train running at constant velocity of 36km/h

(c) just after it is dropped from the window of a train accelerated with 1m/s^2

(d) lying on the floor of a train which is accelerating with 1 m/s^2

Solution:

(a) 1N vertically downward. (as $F = mg = 0.1 \times 10 = 1\text{N}$)

(b) 1N vertically downward (train with uniform velocity is an inertial frame).

(c) In the train accelerated with 1m/s^2 , an additional pseudo force, $F_o = ma_o = 0.1 \times 1 = 0.1\text{N}$ acts on the stone in the horizontal direction. But as soon as the stone is dropped F_o no longer acts on it and the net force acting on it is $F = mg = 0.1 \times 10 = 1\text{N}$ vertically downward

(d) 0.1N along the horizontal

Question (3). A force acts for 10 second on a body of mass 10g, after which the force ceases to act and the body describes 50 cm in next 5second. Find the magnitude of the force

Solution:

The body describes 50cm after the force stops acting and assuming the velocity to be uniform, velocity $= \frac{\text{distance}}{\text{time}} = \frac{50}{5} = 10\text{cm/s}$

If a is the acceleration produced in the body, then from $v = u + at$

$a = \frac{v-u}{t} = \frac{10-0}{10} = 1\text{cm/s}^2$

$$F=ma= 10 \times 10^{-3} \times 1 \times 10^{-2} = 1.0 \times 10^{-4} \text{ N}$$

Question (4). A hunter has a machine gun that can fire 50g bullets with a velocity of 900m/s. A 40 kg tiger springs at him with a velocity of 10 m/s . How many bullets must the hunter fire into the tiger in order to stop him in his track

Solution:

Here ,mass of the bullet , $m=50\text{g} = 0.05\text{kg}$

Mass of the tiger, $M = 40\text{kg}$

Velocity of the bullet , $v = 900 \text{ m/s}$

Velocity of the tiger, $V = 10 \text{ m/s}$

To stop the tiger in his track, let n bullets be fired into the tiger.

Clearly, momentum of the tiger = momentum of n bullets

Or, $MV = nmv$

$$\text{Or, } n = \frac{MV}{mv} = \frac{40 \times 10}{0.05 \times 900} = 9$$

Obviously 9 bullets are required to stop the tiger in his track.

Question (5). A shell of mass 0.02kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 m/s , what is the recoil speed of the gun.

Solution:

Mass of the shell, $m = 0.02 \text{ kg}$

Mass of the gun, $M = 100 \text{ kg}$

Speed of the shell, $v = 80 \text{ m/s}$

Let V be recoil speed of the gun

Applying the law of conservation of momentum ,

$$\text{i.e., } mv + MV = 0$$

$$\text{or } V = \frac{-mv}{M} = \frac{0.02 \times 80}{100} \text{ m/s}$$

$$= -0.016 \text{ m/s}$$

As is obvious , negative sign indicates that gun kicks backwards as the bullet moves forward.

Question (6). A 40kg shell is flying at a speed of 72 km/h. It explodes into two pieces. One of the pieces of mass 15kg stops. Calculate the speed of the other.

Solution:

initial momentum of the shell = MV

$$\text{Velocity of the shell in m/s} = 72 \times \frac{5}{18} = 20 \text{ m/s}$$

$$\text{Initial momentum of the shell} = 40 \times 20 = 800 \text{ kgm/s}$$

$$\text{Final momentum} = 15 \times 0 + 25 \times v = 25v$$

$$v = \frac{800}{25} = 32 \text{ m/s.}$$

Question (7). Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary, given that the coefficient of static friction between the box and the train's floor is 0.2.

Solution:

Since the acceleration of the box is due to the static friction,

$$ma = f_s \leq f_s \text{N} = \mu_s mg$$

$$\text{i.e., } a \leq \mu_s g$$

So , $a_{max} = \mu_s g = 0.2 \times 10 = 2.0 \text{ m/s}^2$

Question (8). Eight five rupee coins are put on top of each other on a table. Each coin has a mass m . Give the magnitude and direction of

(a) The force on the fifth coin (counted from the bottom) due to all the coins on the top,

(b) The reaction of the 4th coin on the 5th coin.

Solution:

(a) The force on the 5th coin is due to the weight of the three coins kept above it. Weight of one coin is (mg) . So ,the weight of the three coins are $(3mg)$. Force exerted on the fifth coin is 3 mg N and the force acts vertically downwards.

(b) The 4th coin is under the weight of four coins above it and experiences a downward force due to the four coins. The total downward force on the 4th coin is 4 mg N . So according to the Newton's third law of motion , the 4th coin will exert a reaction force upwards . Therefore , the force exerted by the 4th coin on the 5th coin is equal to 4 mg N and acts in the upward direction

Question (9). A bullet of mass 60 g ,moving with a velocity of 500 m/s , is brought to rest in 0.01 s .Find the impulse and the average force of blow.

Solution:

$$\begin{aligned}\text{Impulse} &= \text{change in momentum} \\ &= \text{final momentum} - \text{initial momentum} \\ &= 60 \times 10^{-3} \times 500 - 0 \\ &= 30 \text{Ns.}\end{aligned}$$

$$\text{Impulse} = \text{impulsive force} \times \text{time}$$

$$\therefore \text{ Impulsive force} = \frac{\text{impulse}}{\text{time}} = \frac{30}{0.01} = 3000 \text{ N}$$

Question (10). A bullet of mass 0.04 kg moving with a speed of 90 m/s enters a heavy wooden block and is stopped after a distance of 60 cm . What is the average resistive force exerted by the block on the bullet?

Solution:

$$\text{From } a = \frac{v^2 - v_0^2}{2s} , \quad a = \frac{0 - (90)^2}{2 \times 0.6} = 6750 \text{ m/s}^2$$

$$\text{Resistive force} = ma = 0.04 \times 6750 = 270 \text{ N}$$

Question (11). A cricket ball of mass 70 g moving with a velocity of 0.5 m/s is stopped by player in 0.5 s . what is the force applied by player to stop the ball?

Solution:

$$F = \frac{m(v-u)}{t} = \frac{0.07(0-0.5)}{0.5} = -0.07 \text{ N}$$

Question (12). How much momentum will a basketball of mass 5 kg transfer to the ground if it falls from a height of 50 cm and does not rebound? Take its downward acceleration to be 10 m/s^2

Solution:

$$\begin{aligned}\text{final velocity } V &= \sqrt{2gh} = \sqrt{2 \times 10 \times 0.5} \\ &= 3.16 \text{ m/s}\end{aligned}$$

$$\text{Momentum transferred} = mv = 5 \times 3.16 = 15.8 \text{ kg m/s}$$

Question (13). Which would require greater force : accelerating a 10 g mass at 5 m/s² or 20 g mass at 2 m/s² ?

Solution:

$$\begin{aligned} \text{In the first case } F_1 &= m_1 a_1 \\ &= 0.01 \times 5 = 0.05 \text{ N} \end{aligned}$$

$$\text{In the second case } F_2 = m_2 a_2 = 0.02 \times 2 = 0.04 \text{ N}$$

We find that more force is required to accelerate 10 g at 5m/s² than accelerating 20 g at 2 m/s²

Question (14). A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble

- (a) during its upwards motion
- (b) during its backward direction .
- (c) at the highest point where it is momentarily at rest

Solution:

- (a) 0.5 N vertically downward
- (b) 0.5 N vertically downward
- (c) 0.5 N vertically downward

Question (15). Find the distance travelled by a body before coming to rest , if it is moving with a speed of 10m/s and the coefficient of friction between the ground and the body is 0.4

Solution:

$$u = 10 \text{ m/s} , \quad v = 0$$

$$\text{Force due to kinetic friction } f_k = \mu_k mg = ma$$

$$\therefore a = \mu g = 0.4 \times 10 = 4 \text{ m/s}^2$$

Since the body is slowing down , it is retarded motion

$$\therefore a = - 4 \text{ m/s}^2$$

$$\text{From equations of motion , } v^2 = u^2 + 2as$$

$$0 = 10^2 - 2 \times 4 \times s$$

$$\text{i.e.; } s = 12.5 \text{ m}$$

CHAPTER 5 - WORK ENERGY AND POWER - PROBLEMS

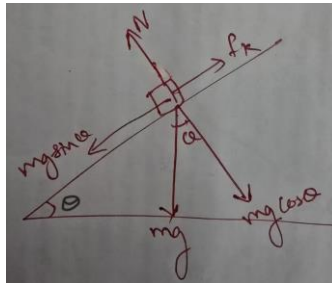
Question (1): A particle moves along the x-axis under a force $F(x) = 3x^2 + 2x$. Find the work done by this force as the particle moves from $x = 0$ to $x = 4$ m.

Solution:

A: Work done by variable force, $W = \int_{x_i}^{x_f} F(x) dx$

$$W = \int_0^4 (3x^2 + 2x) dx = \left[3 * \frac{x^3}{3} + 2 * \frac{x^2}{2} \right]_0^4 = (4^3 + 4^2) - 0 \\ = 64 + 16 = 80 \text{ J}$$

Question(2) A block of mass 4 kg slides down a rough inclined plane of angle 30° and length 10 m. The coefficient of kinetic friction is 0.2. Find the work done by friction?



Solution:

Frictional force, $F_k = \mu_k N = \mu_k mg \cos \theta = 0.2 \times 4 \times 9.8 \times \cos 30^\circ = 0.2 \times 4 \times 9.8 \times 0.866 \approx 6.78 \text{ N}$

Work = $-f_k \times d = -6.78 \times 10 = -67.8 \text{ J}$

Question (3): A block of mass 2 kg moving at 4 m/s compresses a spring of spring constant 800 N/m. Find the maximum compression of the spring.

Solution:

kinetic energy of the block is stored as the potential energy of spring.

So , Kinetic energy = Potential energy

$$\frac{1}{2}mv^2 = \frac{1}{2} kx^2$$

$$\frac{1}{2} \times 2 \times 16 = \frac{1}{2} * 800 * x^2$$

$$x^2 = 2 \times 16 / 800 = 0.04$$

$$x = \sqrt{0.04} = 0.2 \text{ m}$$

Question (4): 5 kg object is acted upon by a constant force of 30 N at 60° to the horizontal. It moves 4 m along a frictionless surface. Find the final speed.

Solution:

$$W = F d \cos \phi = 30 \times 4 \times \cos 60^\circ = 60 \text{ J}$$

Using work-energy theorem:

$$W = \Delta K.E$$

$$60 = \frac{1}{2}mv^2 - 0$$

$$60 = \frac{1}{2} \times 5 \times v^2$$

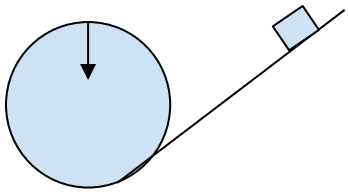
$$v^2 = 120/5 = 24$$

$$v = \sqrt{24} = 4.9 \text{ m/s}$$

Question (5) A block slides from a height h and enters a loop of radius R . Find the minimum height so that the block just completes the loop.

Solution:

At the top of the loop:
centripetal force is provided by Gravitational force and Normal reaction.



For minimum speed, N should be zero

$$\text{So } mv^2/R = mg$$

$$v^2 = Rg$$

Total energy at start = total energy at top of loop

$$mgh = mg(2R) + \frac{1}{2}mv^2$$

$$mgh = mg2R + \frac{1}{2}mRg = mg(2R + \frac{1}{2}R)$$

$$mgh = \frac{5}{2} mg R$$

$$\text{or } h = \frac{5}{2} R$$

Question (6): A force vs. displacement graph is a triangle, with force linearly increasing from 0 to 10 N over 5 m. Find the work done.

Solution:

work done = Area under the triangle

$$W = \frac{1}{2} \times 5 \times 10 = 25 \text{ J}$$

Question (7) A 2 kg block is pushed with 10 J energy across a rough surface and stops after 2 m. Find μ_k .

Solution:

Energy given to block is equal to work done by friction to stop the block

$$10 \text{ J} = \text{force} \times \text{displacement}$$

$$10 = \mu_k \times N \times \text{displacement}$$

$$10 = \mu_k \times 2 \times 9.8 \times 2$$

$$\mu_k = 10/39.2 = 0.255$$

Question (8) An engine pumps up 100kg of water to a height of 10m in 5 seconds. If the efficiency of the engine is 60%, calculate the power of the engine in kW. (take $g=10\text{m/s}^2$)

Solution:

$$\begin{aligned}\text{power required, } P &= \frac{W}{t} \\ &= mgh/t = 100 \times 10 \times 10 / 5 \\ &= 2000 \text{ watts}\end{aligned}$$

Efficiency of the Engine = 60%

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} \times 100 = 60$$

$$\begin{aligned}\text{Input power} &= \text{output power} \times 100 / 60 \\ &= 2000 \times 100 / 60 = 3333 \text{ watts} = 3.333 \text{ kW}\end{aligned}$$

Question (9): A bullet of mass 20g and velocity 500m/s is passed through a mud wall of thickness 80 cm. Its velocity reduces to 50m/s. Find the average resistance offered by the wall.

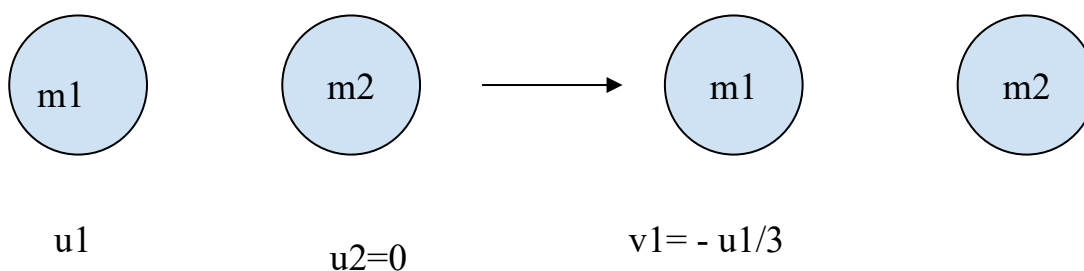
Solution:

from work energy theorem, work done = change in kinetic energy

$$\begin{aligned}F \times \text{displacement} &= K.E_f - K.E_i \\ F \times 1\text{m} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 10 \times 10^{-3} \times 50^2 - \frac{1}{2} \times 10 \times 10^{-3} \times 500^2 \\ F &= -1237.5 \text{ N} \\ \text{Resistive force is } 1237.5 \text{ N}\end{aligned}$$

Question (10) A ball of 0.1 kg makes an elastic head on collision with a ball of unknown mass that is initially at rest. If the 0.1 kg ball rebounds at one third of its original speed, what is the mass of the other ball ?

Solution:



$$V1 = \left[\frac{m1-m2}{m1+m2} \right] u1$$

$$\frac{-u1}{3} = \left[\frac{0.1-m2}{0.1+m2} \right] u1$$

$$-(0.1+m2)=3 (0.1-m2)$$

$$-0.1 - m_2 = 0.3 - 3m_2$$

$$3m_2 - m_2 = 0.3 + 0.1 = 0.4$$

$$2m_2 = 0.4 \quad \text{or } m_2 = 0.4/2 = 0.2$$

Mass of the other ball is 0.2kg

6 SYSTEMS OF PARTICLES AND ROTATIONAL MOTION

Question (1): In the HCl molecule, the separation between the nuclei of the two atoms is about $1.27 \text{ \AA}$. Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

Solution:

Let us take hydrogen atom as the origin.

If x_1 , x_2 and x_{cm} are the distances of hydrogen atom, chlorine atom and the CM of the molecule from this origin, then

$$x_{cm} = \frac{(m_1 x_1 + m_2 x_2)}{m_1 + m_2}$$

Here m_1 and m_2 are the masses of hydrogen atom and the chlorine atom respectively,

Then $m_1 = 1$, $m_2 = 35.5$, Also $x_1 = 0$, $x_2 = 1.27 \text{ \AA}$

$$\text{Thus, } x_{cm} = \frac{(1 \times 0 + 35.5 \times 1.27)}{1 + 35.5} = 1.24 \text{ \AA}$$

Question (2): The angular velocity of an engine increases from 1200 rpm to 3000 rpm in 10s. Compute (a) angular acceleration and (b) the number of revolutions made during this time.

Solution:

$$(a) \omega_0 = 2\pi f = 2\pi \times \frac{1200}{60} = 40\pi \frac{\text{rad}}{\text{s}}$$

$$\omega = 2\pi \frac{3000}{60} = 100\pi \text{ rad/s}$$

$$\text{Angular acceleration, } \alpha = \frac{\omega - \omega_0}{10} = \frac{100\pi - 40\pi}{10} \text{ rad/s}^2 = 6\pi \text{ rad/s}^2$$

$$(b) \text{ From } 2\alpha\theta = \omega^2 - \omega_0^2 \text{ or from } \theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = 700\pi$$

$$\text{Number of revolutions} = \frac{\theta}{2\pi} = \frac{700\pi}{2\pi} = 350$$

Question (3): A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley manner, what is the speed of the CM of the (trolley + child) system?

Solution:

when the child gets up and runs about on the trolley, the speed of the centre of mass of the trolley and child remains unchanged irrespective of the manner of motion of the child. It is because here child and trolley constitute one single

system and forces involved are purely internal forces. As there is no external force, there is no change in momentum of the system and velocity remains unchanged.

Question (4): A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad/s. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis.?

Solution:

$M = 20 \text{ kg}$ Type equation here.

Angular speed $\omega = 100 \text{ rad/s}$; $R = 0.25 \text{ m}$

Moment of inertia of the cylinder about its axis, $I = \frac{MR^2}{2} = \frac{20 \times 0.25^2}{2} = 0.625 \text{ kgm}^2$

Rotational KE, $E = \frac{1}{2} I \omega^2 = \frac{1}{2} \times 0.625 \times 100^2 = 3125 \text{ J}$

Question (5): A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? what is the linear acceleration of the rope? Assume that there is no slipping.

Solution:

Here, $M = 3 \text{ kg}$, $R = 40 \text{ cm} = 0.4 \text{ m}$

Moment of inertia of the hollow cylinder about its axis, $I = MR^2 = 3 \times 0.4^2 = 0.48 \text{ kgm}^2$

Force applied $F = 30 \text{ N}$

Torque, $\tau = R \times F = 0.4 \times 30 = 12 \text{ N-m}$

If α is angular acceleration produced, then from $\tau = I\alpha$

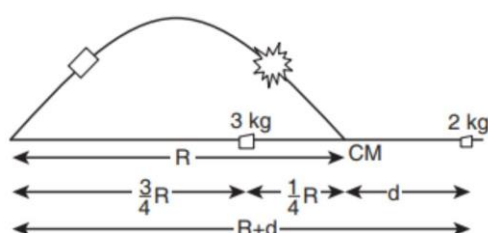
$$\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 \text{ rad s}^{-2}$$

Linear acceleration, $a = R\alpha = 0.4 \times 25 = 10 \text{ ms}^{-2}$

Question (6): A projectile of mass 5 kg, in its course of motion explodes on its own into two fragments. One fragment of mass 3 kg falls at three fourth of the range R of the projectile. Where will the other fragment fall?

Solution:

It is an explosion of its own without any external influence. After the explosion, the centre of mass of the projectile will continue to complete the parabolic path even though the fragments are not following the same parabolic path. After the fragments have fallen on the ground, the centre of mass rests at a distance R (the Range) from the point of projection as shown in the diagram.



If the origin is fixed to the final position of the centre of mass, the principle of moments holds good i.e ; $m_1 x_1 = m_2 x_2$

Where $m_1 = 3 \text{ kg}$, $m_2 = 2 \text{ kg}$, $x_1 = \frac{1}{4} R$. The value of $x_1 = \frac{1}{4} R$. The value of $x_2 = d$

$$3 \times \frac{1}{4} R = 2 \times d$$

$$d = \frac{3}{8} R$$

The distance between the point of launching and the position of 2kg mass is $R + d$

$$R + d = R + \frac{3}{8} R = \frac{11}{8} R = 1.375 R$$

The other fragment falls at a distance of $1.375R$ from the point of launching. (Here R is the range of the projectile)

Question (7): A sphere of radius 10 cm weighs 1 kg. Calculate its moment of inertia: (i) about the diameter (ii) about the tangent.

Solution:

Here mass of the sphere, $M = 1 \text{ kg}$, radius of the sphere, $R = 100 \text{ cm} = 0.1 \text{ m}$

We know that moment of inertia of the sphere about its tangent $= \frac{1}{2} MR^2$

$$= \frac{1}{2} \times 1 \times 0.1^2 = 4 \times 10^{-3} \text{ kg m}^2$$

Further moment of inertia of the sphere about its tangent $= \frac{7}{5} MR^2$

Substituting the values $MI = 1.4 \times 10^{-2} \text{ kg m}^2$

Question (8): A solid cylinder rolls down an inclined plane. Its mass is 2 kg and radius 0.1 m. If the height of the inclined plane is 4 m, what is its rotational KE when it reaches the foot of the plane. Assume that the surfaces are smooth.

Solution:

Let m be the mass of the cylinder and r be its radius. Let it falls through a height h .

Here $m = 2 \text{ kg}$, $h = 4 \text{ m}$

MI of the cylinder about its own axis, i.e., $I = \frac{1}{2} mr^2$

If v is the velocity of the cylinder at the end of the inclined plane, $v^2 = 2gh$

$$\begin{aligned} \text{Rotational KE} &= \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{1}{2} mr^2 \right) \omega^2 = \frac{1}{4} mr^2 \omega^2 = \frac{1}{4} m v^2 = \frac{1}{4} m (2gh) = \frac{1}{2} mgh \\ &= \frac{1}{2} \times 2 \times 9.8 \times 4 \\ &= 39.2 \text{ J} \end{aligned}$$

Question (9): Find the angular momentum of a car of mass 1500 kg moving in a circular track of radius 50 m with a speed of 40 m/s (relative to the centre of the track)

Solution:

Angular momentum, $L = mvr$

$$= 1500 \times 40 \times 50 = 3.0 \times 10^6 \text{ m}^2/\text{s}$$

Question (10): A solid sphere of radius 0.50 m and mass 15 kg rotates in the X-Y plane about an axis through its centre. Find the angular momentum when the angular velocity is 3 rad/s

Solution:

$$\text{Here } I = \frac{2}{5} MR^2 = \frac{2}{5} \times 15 \times 0.5^2 = 1.5 \text{ kgm}^2$$

$$\text{Angular momentum, } L = I\omega = 1.5 \times 1.5 = 4.5 \text{ kgm}^2\text{s}$$

For anticlockwise rotation, $\vec{L}$ is along positive Z direction and for clockwise rotation, $\vec{L}$ is along negative Z direction.

Question (11): If the earth suddenly contracts to half its present radius, by how much would the day be decreased, supposing the earth to be a sphere.?

Solution:

According to the law of conservation of angular momentum,

$$I_1 \omega_1 = I_2 \omega_2$$
$$\left(\frac{2}{5}MR^2\right)\frac{2\pi}{24} = \frac{2}{5}M\left(\frac{R}{2}\right)^2 \times \frac{2\pi}{t}$$

Where t is the new length of the day in hours. As t = 6h the day would decrease by 18h

Question (12): Find the torque of a force (7i + 3j - 5k) about the origin. The force acts on a particle whose position vector is (i - j + k).

Solution:

$$\vec{F} = (7\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}), \vec{r} = (\mathbf{i} - \mathbf{j} + \mathbf{k})$$

$$\text{Torque } \tau = \vec{r} \times \vec{F}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix}$$
$$= 2\mathbf{i} + 12\mathbf{j} + 10\mathbf{k}$$

Question (13): To maintain a rotor at a uniform angular speed of 200 s^{-1} , an engine needs to transmit a torque of 180 Nm. What is the power required by the engine? Assume the engine is 100% efficient.

Solution:

$$\text{Speed of rotation of the rotor, } \omega = 200 \text{ s}^{-1}$$

$$\text{Torque acting on the engine, } \tau = 180 \text{ Nm}$$

$$\text{If the power of the engine required is P, } P = \tau\omega = (180 \times 200) \text{ W} = 36000 \text{ W} = 36 \text{ Kw}$$

Question (14): A sphere of mass 10 kg rolls on a flat surface without slip with a speed of 50 cm/s. Calculate the total energy in joule.

Solution:

since the sphere rolls without slip, its total energy (K) is the sum of its kinetic energy of translation and kinetic energy of rotation, i.e.,

$$K = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \omega^2$$
$$K = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} \left(\frac{2}{5} MR^2\right) \omega^2$$

$$\text{As } v_{cm} = R\omega \quad K = \frac{7}{10} M v_{cm}^2 = \frac{7}{10} \times 10 \times 0.5^2 = 1.75 \text{ J}$$

7 GRAVITATION

Question (1): Distance of the moon from the earth is 3.84×10^8 m and its time period is 27.3 days. Obtain the mass of the earth.

Solution:

If M is the mass of the earth and r is the distance of the moon from the earth, then according to Kepler's third law,

$$T^2 \propto r^3$$

$$T^2 = (4\pi^2/GM) \times r^3$$

$$\text{ie., } M = 4\pi^2 r^3 / GT^2$$

$$M = 4 \times 3.14^2 \times (3.84 \times 10^8)^3 / 6.67 \times 10^{-11} (27.3 \times 24 \times 60 \times 60)^2$$
$$= 6.02 \times 10^{24} \text{ Kg.}$$

Question (2): A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.5×10^9 Km away from the sun?

Solution:

according to Kepler's third law,

$$T^2 \propto r^3$$

$$T_E^2 \propto r_E^3 \quad (1)$$

$$T_S^2 \propto r_S^3 \quad (2)$$

$$(2)/(1) = T_S^2 / T_E^2 = r_S^3 / r_E^3$$
$$= (29.5/1)^2 = r_S^3 / (1.5 \times 10^9)^3$$
$$= r_S^3 = (29.5)^2 \times (1.5 \times 10^9)^3$$
$$r_S = 1.432 \times 10^{13} \text{ m}$$

Question (3): If the mass of one particle is increased by 50% and the mass of another particle is decreased by 50%, is there any change in gravitational force between them ?

Solution:

Yes. Decreases by 25%.

Calculations:- by the universal law of gravitation, $F = GMm/R^2$

$$F_1 = [G(M + M \times 50/100)(m - m \times 50/100)]/R^2$$

$$[GMm/R^2 - GMm \times 50/100 + GmM \times 50/100 - GMm \times 50 \times 50 / 10000]/R^2$$

$$[GMm/R^2 - GMm \times 25/100]/R^2$$

F - 25% of F , ie, force decreases by 25%

Question (4): The centres of two identical spheres are at a distance one metre apart. If the gravitational force between them is 1N, then find the mass of each sphere. ($G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$)

Solution:

$$\text{gravitational force } F = GMM/R^2$$

$$M^2 = FR^2/G$$

Substituting the values, we get $M = 1.224 \times 10^5 \text{ kg}$

Question (5): Two masses M and $4M$ are kept at a distance of 12m apart. A small particle of mass m is to be placed so that the net gravitational force on it is zero. What will be its distance from mass M

Solution:

Force on m by M = Force on m by 4M

$$GM_1 m/r^2 = GM_2 m/(d-r)^2$$

$$[(d-r)/r]^2 = M_2/M_1$$

$$[(d/r)-1]^2 = M_2/M_1$$

$$M_1 = M \text{ And } M_2 = 4M$$

$$\begin{aligned} r &= d[\sqrt{M}/(\sqrt{M} + \sqrt{4M})] \\ &= 12[\sqrt{M}/(\sqrt{M} + \sqrt{4M})] \\ &= 12 \times 1/3 = 4m \end{aligned}$$

Question (6): A mass M is split into m and M-m, which are separated by a certain distance. What ratio of m/M maximises the gravitational force between them?

Solution:

If r is the distance between m and M-m, then the gravitational force between them will be ,

$$\begin{aligned} F &= Gm(M-m)/r^2 \\ &= G(Mm-m^2)/r^2 \end{aligned}$$

For F to be maximum $dF/dm = 0$ and r is constant

$$G/r^2 \times d(Mm-m^2)/dm = 0 \quad \text{therefore} \quad d(Mm-m^2)/dm = 0$$

$$M-2m = 0$$

$$m/M = 1/2$$

Question (7): A body weighs 63N on the surface of the earth. What is the gravitational force on it at a height equal to half the radius of the earth ?

Solution:

On the surface of the earth $g = GM/R^2$

At a height h above the earth $g' = GM/(R+h)^2$

$$g/g' = R^2/(R+h)^2 = R^2/(R+R/2)^2 = 4/9$$

That is; $g' = 4g/9$

Then, gravitational force at the height, $mg' = m \times 4g/9 = mg(4/9) = 63 \times (4/9) = 28N$.

Question (8): An earth satellite makes a complete circuit around the earth in 90 minutes. If the orbit is circular, calculate the height of the satellite above the earth. Given that the radius of earth = 6370km and $g = 9.8m/s^2$

Solution:

$$T = 90 \text{ Min} = 90 \times 60 = 5400s$$

$$R = 6370km = 6370 \times 10^3 m, \quad g = 9.8 m/s^2 \quad h = ?$$

$$GMm/r^2 = ma = m\omega^2 = mr(2\pi/T)^2 = 4\pi^2 mr / T^2$$

$$.r^3 = GMT^2/4\pi^2 \quad ; \quad r = (GMT^2/4\pi^2)^{1/3} = [g R^2 T^2 / 4\pi^2]^{1/3}$$

$$.r = [(9.8 \times (6370 \times 10^3)^2 \times 5400^2) / 4\pi^2]^{1/3} = 6649 \times 10^3 m = 6649m$$

$$.r = R+h = 6649km; \quad h = 6649-6370 = 279km$$

Question (9): Acceleration due to gravity on earth changes with depth and height. Find the height at which acceleration due to gravity is $1/4^{\text{th}}$ that the surface of the earth.

Solution:

$$g' = g (1 - (2h/R))$$

$$g/4 = g (1 - (2h/R))$$

$$1/4 = 1 - 2h/R$$

$$2h/R = 1 - 1/4 = 3/4 \quad \text{therefore } h = 3/4 \times R/2 = 3/4 \times (6400/2) = 2400\text{km.}$$

Question (10): An artificial satellite of mass 100 kg is in a circular orbit at 500 km above earth's surface. Taking the radius of earth as $6.5 \times 10^6 \text{ m}$. find (a) the acceleration due to gravity at any point along the satellite path. (b) centripetal acceleration of the satellite ?

Solution:

$$h = 500\text{km} \quad R+h = 6.5 \times 10^6 + 0.5 \times 10^6 = 7 \times 10^6 \text{ m}$$

$$g = ? ; a = ?$$

$$\text{a) } g' = g[R/(R+h)]^2 = 9.8 \times (6.5 \times 10^6 / 7.0 \times 10^6)^2 = 8.45 \text{ ms}^{-2}$$

$$\text{b) } a = v^2/r = [gR^2 / (R+h)] / [R+h] = R^2g / (R+h)^2 \\ = (6.5 \times 10^6)^2 \times 9.8 / (7.0 \times 10^6)^2 = 8.45 \text{ ms}^{-2}$$

Question (11): The acceleration due to gravity on the moon's surface is 1.7 m/s^2 . The radius of the moon is 0.27 times that of the earth. Acceleration due to gravity on the surface of the earth is 9.8 m/s^2 . Find the ratio of the mass of the earth to that of the moon.

Solution:

$$\text{for the earth } g_e = GM_e/R_e^2$$

$$\text{For the moon } g_m = GM_m/R_m^2$$

$$\text{Simplifying to, } M_e/M_m = g_e/g_m \times (R_e/R_m)^2$$

$$= 9.8/1.7 \times (1/0.27)^2$$

$$= 77.27$$

Question (12): Period of Saturn is 29.5 earth years. Calculate the average distance of Saturn from the sun. given that the earth's orbit is $1.5 \times 10^8 \text{ km}$.

Solution:

$$(T_1/T_2)^2 = (d_1/d_2)^3$$

$$d_1 = 1.5 \times 10^8 \quad d_2 = ?$$

$$\text{We can simplify the equation and write as, } d_2^3 = T_2^2 d_1^3 / T_1^2$$

$$= (1.5 \times 10^8)^3 \times 29.5^2$$

$$\text{Therefore, } d_2 = 1.5 \times 10^8 \times 29.5^{2/3} = 14.3 \times 10^8 \text{ km}$$

Question (13): A satellite of the earth revolves in circular orbits at a height 250 km above the earth's surface. What is the period of revolution of the satellite?
 $R = 6.38 \times 10^6 \text{ m}$, $g = 9.8 \text{ m/s}^2$

Solution:

$$v = (R^2 g / r) \quad T = 2 \pi r / v = 2 \pi r / R(r/g)^{1/2} = 2 \pi / R (r^3/g)^{1/2}$$
$$R = 6.38 \times 10^6 \text{ m} \quad h = 0.25 \times 10^6 \quad r = R + h = 6.63 \times 10^6 \text{ m}$$
$$\text{Therefore, } T = 2 \pi / 6.38 \times 10^6 [(6.63 \times 10^6) / 9.8]^{1/2} = 5365 \text{ s}$$

Question (14): The mean orbital radius of the Earth around the sun is 1.5×10^8 km. the period of revolution of the earth is 365 days' = $6.7 \times 10^{11} \text{ Nm}^2 \text{ kg}^{-2}$. Estimate the mas of the sun.

Solution:

$$GM_s m_e / r^2 = m_e r \omega^2 = m_e r (2 \pi / T)^2$$
$$M_s = 4 \pi^2 r^3 / G T^2 = 4 \pi^2 (1.5 \times 10^{11}) / 6.7 \times 10^{-11} (365 \times 2 \times 60 \times 60)^2$$
$$= 2 \times 10^{30} \text{ kg.}$$

8.MECHANICAL PROPERTIES OF SOLIDS

Question (1): One end of a rope of 4.5 m and diameter 6mm is fixed to the branch of a tree .A monkey weighing 100N jumps to catch the free end and stays there. Find the elongation of the rope (Young's Modulus = $4.8 \times 10^{11} \text{ N/m}^2$)

Solution:

$$Y = FL / A \Delta l$$
$$\Delta l = F L / Y A$$
$$\Delta l = (100 \times 4.5) / 4.8 \times 10^{11} \times 3.14 \times (3 \times 10^{-3})^2$$

Question (2): To what depth below the surface of sea should a rubber ball be taken so as to decrease its volume by 0.1 % (density of sea water = 1000 kg/m^3 , Bulk modulus of rubber = $9 \times 10^8 \text{ N/m}^2$, $g = 10 \text{ m/s}^2$)

Solution:

$$\text{Fractional Change in volume } dv/v = 0.1 \% = 0.001$$
$$B = dp / dv/V$$
$$dp = 9 \times 10^5 = h \rho g$$
$$h = 90 \text{ m}$$

Question (3): Young's modulus of Aluminium is $70 \times 10^9 \text{ Nm}^2$ and that of copper is $120 \times 10^9 \text{ Nm}^2$. Same strain is to be produced on an aluminium wire and a copper wire of equal cross section .Which wire requires more force ?

Solution:

$$\text{a) } Y = \text{Stress} / \text{Strain} = 150 \times 10^6 / 0.002$$
$$= 7.5 \times 10^{10} \text{ Nm}^{-2}$$

$$\text{b) } Y = \text{Stress} / \text{strain} = (F/A) / \text{Strain}$$

For the same strain and area of cross -section , Y directly proportional to F .So copper wire requires more force.

Question (4): A structural steel rod has a radius of 10 mm and a length of 1.0 m .A 100KN force stretches it along its length Calculate i) Stress ii) Elongation and iii) Strain (Young's modulus of steel is $2.0 \times 10^{11} \text{ N/m}^2$)

Solution:

$$\text{Area} = 3.14 r^2$$

$$\text{Stress} = F/A = 100000/3.14 \times 10^{-4} = 3.18 \times 10^8 \text{ Pa}$$

$$\text{Strain} = \text{Stress} / Y = 1.59 \times 10^{-3}$$

$$\text{Elongation} = \text{Strain} \times \text{Length} = 1.59 \times 10^{-3} \times 1.0 = 1.59 \text{ mm}$$

Question (5): The young's modulus of the material of a wire is Y .If the radius of the wire is doubled ,the young's modulus of the new wire will be ,, ,, ,, ,, ,, ,, ,, ,, ,, ,,

a) Y b) 2Y c) Y/2 d) Y/4

Solution:

$$Y \quad \text{stress} = Y \Delta l/l = Y x l/l$$

Question (6): Two wire A and B of the same material have their lengths in the ratio 1:2 and diameters in the ratio 2:1 .if they are stretched by the same force ,the ratio of their elongations is

Solution:

$$dl_1 = (F \times 1) / (Y \times (3.14 \times 2^2)) , dl_2 = (F \times 2) / (Y \times (3.14 \times 1^2))$$

$$dl_1 / dl_2 = 1/8$$

Question (7): A steel wire has a radius of 10 mm and a length of 1 m. A 100 KN force stretches it along its length. Calculate the stress and elongation on the wire. Young's modulus of steel is $2 \times 10^{11} \text{ Nm}^{-2}$.

Solution:

$$F/A = Y(\Delta l/l)$$

$$\Delta l = 1.59 \text{ mm}$$

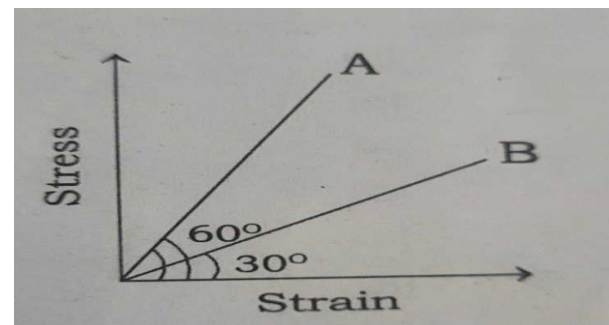
Question (8): With in proportional limit , the stress versus strain graphs for wire of two materials A and B are given .Then their Youngs Modulus Y_A and Y_B is related as

Solution:

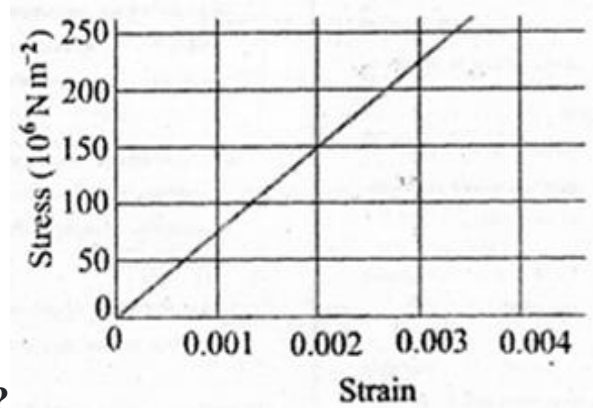
Slope of the graph gives the young's modulus

$$Y_A/Y_B = \tan 60^\circ / \tan 30^\circ = 3$$

$$Y_A = 3Y_B$$



Question (9): Figure shows the strain stress curve for a material. What is the



young's modulus of the material.?

Solution:

slope $7.5 \times 10^{10} \text{ N/m}^2$

Question (10): A steel wire of length 1.5 m and diameter 25 cm is loaded with a force of 98 N. The increase in the length of the wire is $1.5 \times 10^{-4} \text{ m}$. Calculate the tensile stress and the fractional change in length of the wire.

Solution:

Stress = F/A Tensile stress $1.99 \times 10^7 \text{ Nm}^{-2}$ $\Delta l/l = 10^{-4}$

Question (11): A metal cube of side 10 cm is subjected to a shear stress 10^4 N/m^2 . Calculate the rigidity modulus, if the top of the cube is displaced by 0.05 cm with respect to its bottom.

Solution:

$G = Fl / Ax = 0.2 \times 10^7 \text{ Nm}^{-2}$

9 MECHANICAL PROPERTIES OF FLUIDS

Question (1): .An elephant of weight 25,600N stands on its four feet. Find the pressure exerted on the ground if the area of each foot is approximately 250 cm^2 . Give your answer in SI unit.

Solution:

Force = 25,600 N

Area of 1 foot = 250 cm^2

Area of 4 feet = $250 \times 4 = 1000 \text{ cm}^2 = 1000 \times 10^{-4} \text{ m}^2$
 $= 0.1 \text{ m}^2$

Pressure = $\frac{\text{Normal force}}{\text{Area}}$
 $= \frac{25,600}{0.1} = 256,000 \text{ Pa}$

Question (2): The gauge pressure at the bottom of a water tank is 2.7kPa. Find the height of the water. (Use $g=10\text{m/s}^2$)

Solution:

Given:

Gauge pressure $P_{\text{gauge}} = 2.7 \text{ kPa} = 2700 \text{ Pa}$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

$$g = 10 \text{ m/s}^2$$

$$P_{\text{gauge}} = \rho g h$$

$$h = \frac{P_{\text{gauge}}}{\rho g}$$

$$h = \frac{2700}{1000 \times 10} = 0.27 \text{ m}$$

Question (3): A hydraulic lift is used to lift a car of mass 3000 kg. The area of the piston carrying the load is 425cm^2 . What is the maximum pressure the smaller piston has to bear?

Solution:

Given: $m = 3000 \text{ kg}$

$$A = 425 \text{ cm}^2 = 425 \times 10^{-4} \text{ m}^2$$

Force on the large piston:

$$F = mg = 3000 \times 9.8 = 29,400 \text{ N}$$

Maximum pressure on the large piston:

$$P = \frac{F}{A} = \frac{29,400}{425 \times 10^{-4}} \\ = 6.917 \times 10^5 \text{ Pa}$$

By Pascal's law, the pressure is transmitted equally. Therefore, the pressure on the smaller piston is $6.917 \times 10^5 \text{ Pa}$

Question (4): What is the pressure on a scuba diver at a depth of 30 m.

Solution:

$$P = P_a + \rho g h \\ = 1.01 \times 10^5 + 1000 \times 9.8 \times 30 \\ = 1.01 \times 10^5 + 2.94 \times 10^5 \\ = 3.95 \times 10^5 \text{ N/m}^2$$

Question (5): At a depth of 1000 m in an ocean (a) what is the absolute pressure? (b) What is the gauge pressure? (c) Find the force acting on the window of area $20 \text{ cm} \times 20 \text{ cm}$ of a submarine at this depth, the interior of which is maintained at sea-level atmospheric pressure. (The density of sea water is $1.03 \times 10^3 \text{ kg m}^{-3}$, $g = 10 \text{ m s}^{-2}$)

Solution:

Here $h = 1000 \text{ m}$

and $\rho = 1.03 \times 10^3 \text{ kg/m}^3$

(a) absolute pressure

$$\begin{aligned}P &= P_a + \rho gh \\&= 1.01 \times 10^5 \text{ Pa} + 1.03 \times 10^3 \text{ kg/m}^3 \times 10 \text{ m/s}^2 \times 1000 \text{ m} \\&= 104.01 \times 10^5 \text{ Pa} \\&\approx 104 \text{ atm}\end{aligned}$$

(b) Gauge pressure is

$$\begin{aligned}P - P_a &= \rho gh = P_g \\P_g &= 1.03 \times 10^3 \text{ kg/m}^3 \times 10 \text{ m/s}^2 \times 1000 \text{ m} \\&= 103 \times 10^5 \text{ Pa} \approx 103 \text{ atm}\end{aligned}$$

(c) The pressure outside the submarine is

$P = P_a + \rho gh$ and the pressure inside it is P_a .

Hence, the net pressure acting on the window is gauge pressure, $P_g = \rho gh$.

Since the area of the window is $A = 0.04 \text{ m}^2$, the force acting on it is

$$\begin{aligned}F &= P_g A = 103 \times 10^5 \text{ Pa} \times 0.04 \text{ m}^2 \\&= 4.12 \times 10^5 \text{ N}\end{aligned}$$

Question (6): The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20°C is 6.5 cm/s. Compute the viscosity of the oil at 20°C . Density of oil is $1.5 \times 10^3 \text{ kg/m}^3$, density of copper is $8.9 \times 10^3 \text{ kg/m}^3$.

Solution:

$$\begin{aligned}V_t &= \frac{2a^2(\rho - \sigma)g}{9\eta} \\ \eta &= \frac{2a^2(\rho - \sigma)g}{9V_t} \\ \eta &= \frac{2 \times (2 \times 10^{-3})^2 \times (8.9 \times 10^3 - 1.5 \times 10^3) \times 9.8}{9 \times 6.5 \times 10^{-2}} \\ &= 9.9 \times 10^{-1} \text{ kg m}^{-1} \text{ s}^{-1}\end{aligned}$$

Question (7): The diameter of ball A is half that of ball B. What will be the ratio of their terminal velocities in water?

Solution:

$$V_t = \frac{2a^2(\rho - \sigma)g}{9\eta}$$

The terminal velocity is directly proportional to the square of radius of the ball,

If the radius of the ball B is 'r' then,

$$\frac{V_{t,A}}{V_{t,B}} = \frac{(r/2)^2}{r^2} = \frac{1}{4}$$

Therefore the ratio of terminal velocities will be 1:4.

Question (8): In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m/s and 63 m/s respectively. What is the lift on the wing if its area is 2.5 m²? Take the density of air to be 1.3 kg/m³.

Solution:

By Bernoulli's theorem

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Since $v_2 > v_1$, $P_1 > P_2$

$$\begin{aligned}\Delta P &= (P_1 - P_2) = \frac{1}{2}\rho(v_2^2 - v_1^2) \\ &= \frac{1}{2} \times 1.3 \times (70^2 - 63^2) = 605.15 \text{ Pa}\end{aligned}$$

Lift on the wing

$$F = \Delta P \times A = 605.15 \times 2.5 = 1512.87 \text{ N}$$

Question (9): A garden hose having an internal diameter of 2 cm is connected to a lawn sprinkler that consists of an enclosure with 12 holes each 0.2 cm in diameter. If the water in the hose has a speed 1.2 m/s, at what speed does it leave the sprinkler holes.

Solution

By equation of continuity, $a_1 v_1 = a_2 v_2$

$$\pi R^2 v_1 = 12\pi r^2 v_2$$

$$v_2 = \frac{R^2 v_1}{12r^2}$$

$$= \frac{(10^{-2})^2 \times 1.2}{12 \times (0.1 \times 10^{-2})^2}$$

$$= \frac{10^{-4} \times 1.2}{12 \times 10^{-6}} = 10 \text{ m/s}$$

Question (10): . What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20 °C) is 2.50×10^{-2} N/m? If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atmospheric pressure is 1.01×10^5 Pa).

Solution:

$$\begin{aligned}\text{(i) For soap bubble, excess of pressure} &= \frac{4S}{R} \\ &= \frac{4 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}} = 20 \text{ Pa}\end{aligned}$$

$$\begin{aligned}\text{(ii) For air bubble, excess of pressure} &= \frac{2S}{R} \\ &= \frac{2 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}} = 10 \text{ Pa}\end{aligned}$$

Total pressure inside the air bubble = Atm pressure + hydrostatic Pressure ($h\rho g$) + excess of pressure

$$\text{Relative density} = \frac{\text{density of solution}}{\text{density of water}} = 1.2$$

$$\text{Density of soap solution} = 1.2 \times 1000 = 1200$$

$$\begin{aligned}\text{Total pressure} &= 1.01 \times 10^5 + 0.4 \times 1200 \times 9.8 + 10 \\ &= 1.06 \times 10^5 \text{ Pa}\end{aligned}$$

Question (11): . Calculate the excess pressure inside an air bubble of radius 1 mm formed just below the free surface of water. Given surface tension of water $72 \times 10^{-3} \text{ N/m}^2$

Solution

$$\Delta P = \frac{2S}{R} = \frac{2 \times 72 \times 10^{-3}}{10^{-3}} = 144 \text{ N/m}^2$$

Question (12): Find the capillary rise when a capillary tube of radius 0.05 cm is dipped vertically in water. Surface tension for water is 0.073 N/m, density of water is 1000 kg/m^3 .

Solution

$$h = \frac{2S \cos \theta}{\rho g a}$$

For water-glass angle of contact $\theta = 0$, $\cos 0 = 1$

$$h = \frac{2S}{\rho g a}$$

$$\begin{aligned}h &= \frac{2 \times 0.073}{1000 \times 9.8 \times 0.05 \times 10^{-2}} \\ h &= 2.98 \times 10^{-2} \text{ m}\end{aligned}$$

Question (13): Calculate the work done in breaking a water drop of radius 1 mm to 1000 droplets. Given surface tension of water = $72 \times 10^{-3} \text{ N/m}$

Solution:

Work done = $S \times$ increase in area

Volume of the big drop = $1000 \times$ Volume of small drop

$$\begin{aligned}\frac{4\pi R^3}{3} &= 1000 \times \frac{4\pi r^3}{3} \\ R^3 &= 1000 \times r^3 \\ r &= \frac{R}{10}\end{aligned}$$

Increase in area

$$\begin{aligned}\Delta A &= (1000 \times 4\pi r^2 - 4\pi R^2) \\ &= 36\pi \times 10^{-6}\end{aligned}$$

Therefore

$$\begin{aligned}\text{Work done} &= S \times (1000 \times 4\pi r^2 - 4\pi R^2) \\ &= 72 \times 10^{-3} \times 36\pi \times 10^{-6} \\ &= 8139 \times 10^{-9} \text{ J}\end{aligned}$$

Question (14): . A wire ring of internal radius 3 cm and external radius 3.2 cm is rested on the surface of a liquid and is then raised. An extra pulling force equivalent to the weight of 3.03 g is required before the film breaks. Calculate the surface tension of the liquid.

Solution

When the ring is raised, the liquid film forms along both the inner and outer circumference of the ring. The total length, L , of the film in contact with the wire is the sum of these circumferences

$$L = 2\pi r_1 + 2\pi r_2 = 2\pi(r_1 + r_2)$$

$$\begin{aligned} \text{Surface tension} &= \frac{\text{Force}}{\text{length}} \\ &= \frac{mg}{2\pi(r_1 + r_2)} \\ &= \frac{3.03 \times 10^{-3} \times 9.8}{2\pi(3 + 3.2) \times 10^{-2}} = 76.26 \times 10^{-3} \text{ N/m} \end{aligned}$$

Question (15): . A metal plate of 5 cm x 5 cm rests on a layer of castor oil 1 mm thick. If the coefficient of viscosity of the oil is 1.55 Ns/m², what is the horizontal force required to move the plate with a speed of 2 cm/s?

Solution

Coefficient of viscosity, $\eta = \frac{\text{stress}}{\text{strain rate}}$

$$\begin{aligned} \eta &= \frac{F/A}{v/l} = \frac{Fl}{Av} \\ F &= \frac{Av\eta}{l} \\ &= (5 \times 10^{-2})^2 \times 2 \times 10^{-2} \times 1.55 / 1 \times 10^{-3} \\ &= 77.5 \times 10^{-3} \text{ N} \end{aligned}$$

Question (16): A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit?

Solution

The pressure exerted by the column of water must equal the pressure exerted by the column of methylated spirit at the same horizontal level

$$\begin{aligned} P_{\text{water}} &= P_{\text{spirit}} \\ \rho_w g h_w &= \rho_s g h_s \\ \rho_w h_w &= \rho_s h_s \end{aligned}$$

$$\begin{aligned} \text{Specific gravity of spirit} &= \frac{\rho_s}{\rho_w} = \frac{h_w}{h_s} \\ &= \frac{10}{12.5} = 0.8 \end{aligned}$$

10.THERMAL PROPERTIES OF MATTER

Question(1):The triple point of Neon is 24.57K. Express the temperature in the Celsius and Fahrenheit scales.

Solution

$$\frac{C}{100} = \frac{F-32}{180} = \frac{K-273.15}{100}$$

$$\begin{aligned}C &= K - 273.15 \\&= 24.57 - 273.15 \\&= -248.58 \text{ }^{\circ}\text{C}\end{aligned}$$

$$\begin{aligned}F &= \frac{K-273.15}{100} \times 180 + 32 \\&= \frac{24.57- 273.15}{100} \times 180 + 32\end{aligned}$$

$$F = -415.44 \text{ }^{\circ}\text{F}$$

Question(2):Steel rail 40m long is laid at 15 °C. Calculate the space that must be left between consecutive rails, if the highest temperature attained by them during the year is 35 °C. Coefficient of linear expansion for steel is $1.2 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}$.

Solution

$$\begin{aligned}\Delta l &= l_2 - l_1 = l_1 \alpha_l \Delta T \\&= 40 \times 1.2 \times 10^{-5} \times (35 - 15) \\&= 0.96 \times 10^{-2} \text{ m.}\end{aligned}$$

Question(3):The volume of a block of metal changes by 0.12% , when it is heated through 20°C. What is the coefficient of volume expansion of metal.

Solution

$$\alpha_v = \frac{\Delta V}{V \Delta T}$$

$$\Delta V = \frac{0.12}{100} V$$

$$\alpha_v = \frac{0.12 V}{100 V \times 20}$$

$$= 60 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}$$

Question(4):A copper block of mass 2.5kg is heated in a furnace to a temperature of 500 °C and then placed on a large ice block. What is the maximum amount of ice that can melt. Specific heat of copper = $390 \text{ J kg}^{-1} \text{ K}^{-1}$. Latent heat of fusion of water = $335 \times 10^3 \text{ J/kg}$.

Solution

Mass of Copper $m_1 = 2.5\text{kg}$

Specific heat of copper $s = 390 \text{ J kg}^{-1} \text{ K}^{-1}$

Temperature of furnace $\Delta T = 500^\circ\text{C}$.

Latent heat of fusion $L = 335 \times 10^3 \text{ J/kg}$.

Heat absorbed by copper $Q = m_1 s \Delta T$

If m_2 is the mass of ice melted, when copper block is placed on it.

Heat required for it $Q = m_2 L$ this heat is supplied by the copper block.

$$\text{Thus } m_2 L = m_1 s \Delta T$$

$$\text{Or } m_2 = \frac{m_1 s \Delta T}{L}$$

$$= \frac{2.5 \times 390 \times 500}{335 \times 10^3}$$

$$= 1.455\text{kg}.$$

Question(5): When 0.15 kg of ice at 0°C is mixed with 0.3kg of water at 50°C in a container, the resulting temperature is 6.7°C . Calculate the heat of fusion of ice. (specific heat capacity of water $s = 4186 \text{ J kg}^{-1} \text{ K}^{-1}$.)

Solution

$$\begin{aligned}\text{Heat loss by water} &= m_1 s \Delta T \\ &= 0.3 \times 4186 \times (50 - 6.7) \\ &= 54376.14 \text{ J}.\end{aligned}$$

$$\begin{aligned}\text{Heat required to melt ice} &= m_2 L \\ &= 0.15 \times L\end{aligned}$$

$$\begin{aligned}\text{Heat required to raise temperature of ice} &= m_2 s \Delta T \\ &= 0.15 \times 4186 \times (6.7 - 0) \\ &= 4206.93 \text{ J}\end{aligned}$$

$$\text{Heat loss} = \text{Heat gain}$$

$$54376.14 = 0.15 \times L + 4206.93$$

$$\text{Solving } L = 3.34 \times 10^5 \text{ J/kg}.$$

Question(6): Calculate the rise in temperature of water if it falls from a height of 50m, assuming that all the energy is used in heating the water. (specific heat capacity of water $= 4186 \text{ J kg}^{-1} \text{ K}^{-1}$)

Solution

$$\text{Potential energy} = mgh$$

$$\text{Energy absorbed to heat water} = ms\Delta T$$

Potential energy is converted into heat energy.

$$\text{ie } ms\Delta T = mgh$$

$$\Delta T = gh/s$$

$$= \frac{9.8 \times 50}{4186}$$

$$= 0.12^\circ\text{C}$$

Question(7): A lead bullet strikes a target with a velocity 480m/s. Calculate the rise in temperature of the bullet. (Assuming that the energy is equally shared between bullet and target.). specific heat capacity of lead = $0.03 \text{ cal g}^{-1}^\circ\text{C}^{-1}$.

Solution

$$\text{Velocity of bullet} = 480\text{m/s}$$

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

$$0.03 \text{ cal g}^{-1}^\circ\text{C}^{-1} = 0.03 \times 4186 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$\text{Heat gained by the bullet} = ms\Delta T$$

1/2 of Kinetic energy is converted into heat gained by the bullet.

$$\frac{1}{2} \times \frac{1}{2}mv^2 = ms\Delta T$$

$$\Delta T = \frac{v^2}{4s}$$

$$= \frac{480 \times 480}{4 \times 0.03 \times 4186}$$

$$= 457^\circ\text{C}$$

Question(8): Calculate the amount of heat required to convert 1g ice at 0°C to 1g of steam at 100°C .

Solution

$$\text{mass } m = 1\text{g} = 1 \times 10^{-3} \text{ kg.}$$

$$\text{latent heat of fusion of ice } L_f = 3.33 \times 10^5 \text{ J/kg.}$$

$$\text{Specific heat capacity of water } s = 4186 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$\text{Latent heat of vaporisation of water } L_v = 22.6 \times 10^5 \text{ J/kg.}$$

$$\text{Quantity of heat} = mL_f + ms\Delta T + mL_v$$

$$= 1 \times 10^{-3} \times 3.33 \times 10^5 + 1 \times 10^{-3} \times 4186 \times (100 - 0) + 1 \times 10^{-3} \times 22.6 \times 10^5$$

$$= 3014 \text{ J}$$

Question(9): A slab made of copper has a thickness 25cm and area 90cm². If the temperature on either side of slab are 125 °C and 10 °C calculate the heat conduction rate through the slab.

(Thermal conductivity of copper is 400 W m⁻¹ K⁻¹).

Solution

$$\text{Rate of flow of heat } H = \frac{K A (T_c - T_D)}{L}$$

$$\begin{aligned} \text{Here } A &= 90 \text{ cm}^2 = 90 \times 10^{-4} \text{ m}^2 \\ K &= 400 \text{ W m}^{-1} \text{ K}^{-1} \\ L &= 25 \text{ cm} = 25 \times 10^{-2} \text{ m} \\ T_c &= 125^\circ \text{C} \\ T_D &= 10^\circ \text{C} \end{aligned}$$

$$\begin{aligned} \text{Then } H &= \frac{400 \times 90 \times 10^{-4} \times (125 - 10)}{25 \times 10^{-2}} \\ &= 1656 \text{ J/s.} \end{aligned}$$

Question(10): A body cools from 80 °C to 50 °C in 5 minutes. Calculate the time it takes to cool from 60 °C to 30 °C. Temperature of the surroundings is 20 °C.

Solution

Average of 80 °C and 50 °C is 65 °C.

Difference of this average temperature with surroundings $\Delta T = 65 - 20$
 $= 45^\circ \text{C}$

According to Newton's law of cooling,

Change in temperature / time = K ΔT (K is a constant.)

$$\begin{aligned} \text{Change in temperature} &= 80 - 50 \\ &= 30^\circ \text{C.} \\ \text{Time} &= 5 \text{ minutes.} \end{aligned}$$

$$\text{Therefore, } 30/5 = K \times 45 \dots\dots\dots (a)$$

Average of 60 °C and 30 °C is 45 °C.

Difference of this average temperature with surroundings $\Delta T = 45 - 20$
 $= 25^\circ \text{C}$

Here also Change in temperature / time = $K \Delta T$

$$\begin{aligned}\text{Change in temperature} &= 60 - 30 \\ &= 30^\circ\text{C}.\end{aligned}$$

let the corresponding time = t minutes .

$$\text{Then } 30/t = K \times 25 \dots\dots\dots (b)$$

From (a) & (b)

$$\begin{aligned}t &= \frac{45 \times 30 \times 5}{30 \times 25} \\ &= 9 \text{ minutes.}\end{aligned}$$

11.THERMODYNAMICS

Question(1):The pressure of a gas is increased by 50% at constant temperature. The decrease in volume will be nearest to.

Solution:

If $P_1 = P$, $V_1 = V$

$$P_2 = \frac{3}{2} P , V_2 = ?$$

As temperature is constant

$$P_2 V_2 = P_1 V_1$$

$$V_2 = \frac{P_1 V_1}{P_2} = \frac{PV}{\frac{3}{2}P} = \frac{2}{3} V$$

$$\therefore \text{Decrease in volume} = V - \frac{2}{3} V = \frac{1}{3} V = 33\%$$

Question(2):Two samples of a gas initially at same temperature and pressure are compressed from a volume V to $V/2$. One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater?

Solution:

In an isothermal compression,

$$P_2 = P_1 \frac{V_1}{V_2} = P_1 \left(\frac{V}{V/2} \right) = 2P_1$$

In an adiabatic compression,

$$P_2' = P_1' \left(\frac{V_1}{V_2} \right)^\gamma = P_1 (2)^\gamma$$

$$\text{As } \gamma > 1 \quad \therefore P_2' > P_2$$

Question(3):A cylinder containing one gram molecule of the gas was compressed adiabatically until its temperature rose from 27°C to 97°C . Calculate the work done and heat produced in the gas. ($\gamma = 1.5$).

Solution :

Here, initial temperature,

$$T_1 = 27^\circ\text{C} = 273 + 27 = 300\text{K}$$

Final temperature,

$$T_2 = 97^\circ\text{C} = 273 + 97 = 370\text{K}$$

When a gas is compressed adiabatically, work done on the gas is given by,

$$W = \frac{R}{(1-\gamma)} (T_2 - T_1)$$

$$= \frac{8.3 \times (370 - 300)}{1 - 1.5}$$

$$W = - 11.62 \times 10^2 \text{ J}$$

$\therefore$ Heat produced,

$$H = \frac{W}{J} = \frac{11.62 \times 10^2}{4.2} = 276.7 \text{ cal}$$

Question(4): One mole of an ideal gas is taken in a Carnot engine working between 27°C and 227°C . The useful work done in one cycle is 600J. calculate the ratio of volume of the gas at the end and beginning of the volume of the gas at the end and beginning of the isothermal expansion. Given $R = 8.31 \text{ mole}^{-1}\text{K}^{-1}$

Solution:

Here $T_2 = 27^\circ\text{C} = (27 + 273) \text{ K} = 300\text{K}$

$$T_1 = 227^\circ\text{C} = (227 + 273) \text{ K} = 500 \text{ K}$$

$$W = 600\text{J}, R = 8.31 \text{ J mole}^{-1} \text{ K}^{-1}$$

$$\text{As } W = 2.303 R (T_1 - T_2) \log_{10} \frac{V_2}{V_1}$$

$$\therefore \log_{10} \frac{V_2}{V_1} = \frac{W}{2.303 R (T_1 - T_2)}$$

$$= \frac{600}{2.303 \times 8.31 (500 - 300)}$$

$$= 0.1568$$

$$\frac{V_2}{V_1} = \text{antilog}(0.1568) = 1$$

Question(5): Carnot engine takes in a thousand kilo calories of heat from a reservoir at 625°C , and exhaust it to a sink at 27°C . How much work does it perform? What is the efficiency of the engine?

Solution :

Here $Q_1 = 1000\text{k cal} = 10^6 \text{ cal}$

$$T_1 = 627^\circ\text{C} = 900\text{K}$$

$$T_2 = 27^\circ\text{C} = 300\text{K} ; W = ?$$

$$\text{As } \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$Q_2 = \frac{T_2}{T_1} Q_1 = \frac{300}{900} \times 10^6 \text{ cal}$$

$$= \frac{1}{3} \times 10^6 \text{ cal}$$

Work done/cycle $W = Q_1 - Q_2$

$$= 10^6 - \frac{1}{3} \times 10^6 = \frac{2}{3} \times 10^6 \text{ cal}$$

$$= \frac{2 \times 4.2}{3} \times 10^6 \text{ J} = 2.8 \times 10^6 \text{ J}$$

Also, $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{900}$

$$= \frac{2}{3} = \frac{2}{3} \times 100 = 66.67\%$$

Question(6): Calculate the efficiency of a Carnot engine working between steam point and ice point.

Solution :

Here steam point,

$$T_1 = 100^\circ\text{C} = 100 + 273 = 373$$

Ice point,

$$T_2 = 0^\circ\text{C} = 0 + 273 = 273 \text{ K}$$

As $\eta = 1 - \frac{T_2}{T_1}$

$\therefore \eta = 1 - \frac{273}{373} = \frac{100}{373}$

$$= \frac{100}{373} \times 100\% = 26.81 \%$$

Question(7): A gas is compressed isothermally to half its volume. By what factor does the pressure of the gas increase.

Solution:

For isothermal changes,

$$P_2 V_2 = P_1 V_1$$

$$\frac{P_2}{P_1} = \frac{V_1}{V_2} = \frac{V_1}{\frac{1}{2}V_1} = 2$$

Question(8): A gas is suddenly compressed to $\frac{1}{4}$ th of its original volume.

Calculate the rise in temperature, the original temperature being 27°C and $\gamma = 1.5$.

Solution:

let the initial volume, $V_1 = V$

Final volume, $V_2 = V/4$

Initial temp, $T_1 = 27^\circ\text{C} = 273 + 27 = 300\text{K}$

and $\gamma = 1.5$.

Let T_2 K be final temp. after compression.

Since the change is adiabatic,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = 300 \left(\frac{V}{V/4} \right)^{1.5-1}$$

$$300(4)^{1/2} = 600 - 273 = 327^\circ\text{C}$$

$$\therefore \text{rise in temperature} = 327^\circ\text{C} - 27^\circ\text{C}$$

$$= 300^\circ\text{C}$$

Question(9): A gram molecule of a gas at 127°C expands isothermally until its volume is doubled. Find the amount of work done and heat absorbed.

Solution:

Here temperature of the gas ,

$$T = 273 + 127 = 400 \text{ K}$$

Let initial volume of the gas, $V_1 = V$

Final volume of the gas, $V_2 = 2V$

In an isothermal expansion,

$$\begin{aligned} \text{Work done (W)} &= 2.303 RT \log_{10} \frac{V_2}{V_1} \\ &= 2.303 \times 8.3 \times 400 \log_{10} \frac{2V}{V} \\ &= 2.303 \times 8.3 \times 400 \times 0.3010 \\ &= 2.30 \times 10^3 \text{ Joule} \end{aligned}$$

If H is the amount of heat absorbed ,

$$H = \frac{W}{J} = \frac{2030 \times 10^3}{4.2} = 548 \text{ cal}$$

Question(10): A sample of ideal gas ($\gamma = 1.4$) is heated at constant pressure. If an amount 140 J of heat is supplied to the gas. Find (a) change in internal energy of the gas (b) work done by the gas.

Solution:

Let the sample of gas contain n moles.

At constant pressure, let the state of the gas change from (V_1, T_1) to (V_2, T_2)

Heat supplied, $dQ = n C_p (T_2 - T_1) = 140 \text{ J}$

Change in internal energy of the gas,

$$\begin{aligned} dU &= n C_v (T_2 - T_1) = \frac{n C_v}{C_p} C_p (T_2 - T_1) \\ &= \frac{C_v}{C_p} dQ = \frac{140}{1.4} = 100 \text{ J} \end{aligned}$$

Work done by the gas

$$dW = dQ - dU = 140 - 100 = 40 \text{ J}$$

12. KINETIC THEORY OF GASES

Question 1: Calculate the r.m.s velocity of oxygen molecules at STP, the molecular weight of oxygen being 32?

Solution:

$$M = 32 \text{ g} = 32 \times 10^{-3} \text{ Kg}$$

$$\text{Standard pressure } p = 760 \text{ mm of Hg} = 760 \times 10^{-3} \times 13600 \times 9.8 = 101292.8 \text{ Pa}$$

Molar volume of oxygen = 22.4 litre = $22.4 \times 10^{-3} \text{ m}^3$

$$\begin{aligned}\text{RMS velocity } V_{\text{rms}} &= \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}} \\ &= \sqrt{\frac{3 \times 101325 \times 22.4 \times 10^{-3}}{32 \times 10^{-3}}} = 461.2 \text{ m/s}\end{aligned}$$

Question 2: Estimate the average thermal energy of helium

- (i) at room temperature (27°C)
- (ii) Temperature on the surface of sun (6000 K) and temperature of 10^7 K (given $k = 1.38 \times 10^{-23} \text{ J/mol/K}$)

Solution:

$$\underline{K.E} = \frac{3}{2} kT$$

(i) $T = 27^\circ\text{C} = 300 \text{ K}$

$$\underline{K.E} = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21} \text{ J}$$

(ii) $\underline{K.E} = \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 6000 = 1.24 \times 10^{-19} \text{ J}$

$$\underline{K.E} = \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^7 = 2.07 \times 10^{-16} \text{ J}$$

Question 3: A gas is made up of molecules of hydrogen and oxygen.

- a) Which molecules move faster?
- b) What is the approximate ratio of their speeds?

Solution:

a) At given temperature T , average kinetic energy of a gas molecule is

$$\frac{1}{2} m \underline{v}^2 = \frac{3}{2} KT$$

$$\text{rms velocity } \propto \frac{1}{\sqrt{m}}$$

Since hydrogen molecule has lesser mass than oxygen molecule, hydrogen molecule moves faster.

b) Since in the gas, the molecules of hydrogen and oxygen are at same temperatures,

$$\frac{1}{2} m_H \underline{v_H}^2 = \frac{1}{2} m_O \underline{v_O}^2$$

$$\frac{v_H}{v_O} = \sqrt{\frac{m_H}{m_O}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4$$

The ratio of their speeds is 4 : 1

Question 4: Two vessels of same size are at the same temperature. One of them contain 1 Kg of H₂ gas and the other contain 1 Kg of N₂ gas.

- Which of the vessels contains more molecules?
- Which of the vessels is under greater pressure and why?
- In which vessel is average molecular speed greater? How many times the speed of one is greater than that of other gas molecule?

Solution:

a) since the molecular weight of H₂ < N₂, the no. of moles of H₂ gas in 1 Kg is more than that of N₂ gas. Hence the vessel containing H₂ gas contains more molecules.

b) Pressure $P = \frac{2}{3} \times n \times \underline{K.E}$

Average kinetic energy of the gas molecule at given temperature is independent of the gas.

$$P \propto n$$

Since the cylinder containing H₂ gas contains greater number of molecules per unit volume, pressure exerted by the H₂ gas will be more.

c) $\frac{1}{2} m_H v_H^2 = \frac{1}{2} m_N v_N^2$

$$\frac{\text{rms velocity of } H_2}{\text{rms velocity of } N_2} = \sqrt{\frac{m_N}{m_H}} = \sqrt{\frac{28}{2}} = \sqrt{14}$$

Question 5: Calculate the kinetic energy of one mole of argon at 127°C. Given that N = 6.02 x 10²³ mol⁻¹ and K = 1.381 x 10⁻²³ J/ molecule/K.

Solution:

$$N = 6.02 \times 10^{23} \text{ mol}^{-1}$$

$$K = 1.381 \times 10^{-23} \text{ J/ molecule/K}$$

$$T = 127 + 273 = 400 \text{ K}$$

$$R = NK = 8.314 \text{ J/ mole/K}$$

$$\text{KE of one mole of gas} = \frac{3}{2} RT = \frac{3}{2} \times 8.314 \times 400 = 4988.4 \text{ K}$$

Question 6: Calculate the temperature at which r.m.s velocity of hydrogen will be equal to 8 Km/s. Given $R = 8.314 \text{ J/ mole/K}$

Solution:

$$R = 8.314 \text{ J/ mole/K}$$

$$V_{\text{rms}} = 8 \text{ Km/s} = 8000 \text{ m/s}$$

$$\text{Molecular weight of hydrogen, } M = 2\text{g} = 2 \times 10^{-3} \text{ Kg}$$

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$T = \frac{V_{\text{rms}}^2 \times M}{3R} = \frac{8000^2 \times 2 \times 10^{-3}}{3 \times 8.314} = 5134.38 \text{ K}$$

Question 7: Calculate the number of degrees of freedom for one mole of a diatomic gas at N.T.P?

Solution:

At NTP, the number of degrees of freedom of a diatomic gas $f = 5$

$$\text{The number of degrees of freedom for one mole of a diatomic gas} = N \times f = 6.02 \times 10^{23} \times 5$$

$$= 3.01 \times 10^{24}$$

Question 8: A vessel contains argon gas at 127°C . Find the average kinetic energy of argon molecule, if Boltzmann's constant $K = 1.381 \times 10^{-23} \text{ J/ molecule/K}$

Solution:

$$T = 127 + 273 = 400 \text{ K}$$

$$K = 1.381 \times 10^{-23} \text{ J/ molecule/K}$$

Argon is a mono-atomic gas and the no. of degrees of freedom $f = 3$

$$\text{KE of one molecule of argon gas} = f \times \frac{1}{2} KT$$

$$= 3 \times \frac{1}{2} \times 1.381 \times 10^{-23} \times 400$$
$$= 8.28 \times 10^{-21} \text{ J}$$

Question 9: Calculate the mean free path and collision frequency of air molecules if the number of molecules per cm^3 is 3×10^{19} , the diameter of the molecule is $2 \times 10^{-8} \text{ cm}$ and the average molecular speed = 1 Km/s .

Solution:

$$n = 3 \times 10^{19} / \text{cm}^3 = 3 \times 10^{25} / \text{m}^3$$

$$d = 2 \times 10^{-8} \text{ cm} = 2 \times 10^{-10} \text{ m}$$

$$v = 1 \text{ Km/s} = 1000 \text{ m/s}$$

$$\text{Mean free path } \underline{\lambda} = \frac{1}{\sqrt{2}\pi d^2 n} = \frac{1}{\sqrt{2} \times 3.14 \times (2 \times 10^{-10})^2 \times 3 \times 10^{25}} = 1.876 \times 10^{-7} \text{ m}$$

$$\text{Collision frequency } f = \frac{v}{\underline{\lambda}} = \frac{1000}{1.876 \times 10^{-7}} = 5.33 \times 10^9 / \text{s}$$

Question 10: The molar gas volume at STP is 22.4 litre and avagadro number is $N = 6.02 \times 10^{23} \text{ mol}^{-1}$

- Calculate the number of gas molecules per unit volume.
- Calculate the radius of the molecule, given that mean free path is $1.1 \times 10^{-8} \text{ m}$.

Solution:

$$N = 6.02 \times 10^{23} \text{ mol}^{-1}$$

$$V = 22.4 \text{ litres} = 22.4 \times 10^{-3} \text{ m}^3$$

$$\underline{\lambda} = 1.1 \times 10^{-8} \text{ m}$$

$$n = \frac{N}{V} = \frac{6.02 \times 10^{23}}{22.4 \times 10^{-3}} = 2.69 \times 10^{25} / \text{m}^3$$

$$\text{Mean free path } \underline{\lambda} = \frac{1}{\sqrt{2}\pi d^2 n}$$

$$d^2 = \frac{1}{\sqrt{2}\pi \underline{\lambda} n} = \frac{1}{\sqrt{2} \times 3.14 \times 1.1 \times 10^{-8} \times 2.69 \times 10^{25}} = 7.610 \times 10^{-19} \text{ m}^2$$

$$d = 8.72 \times 10^{-10} \text{ m}$$

$$\text{radius } r = d/2 = 4.36 \times 10^{-10} \text{ m}$$

13.OSCILLATIONS

Question (1): A particle executes SHM with amplitude $A = 5 \text{ cm}$ and angular frequency $\omega = 10 \text{ rad s}^{-1}$.

Find:

- Maximum velocity
- Maximum acceleration

Solution

$$\text{Given: } A = 0.05 \text{ m, } \omega = 10$$

- Maximum velocity:

$$v_{\max} = \omega A = 10 \times 0.05 = 0.5 \text{ m s}^{-1}$$

- Maximum acceleration:

$$a_{\max} = \omega^2 A = 10^2 \times 0.05 = 5 \text{ m s}^{-2}$$

Question (2): A body of mass 2 kg is attached to a spring of force constant 800 N m^{-1} . Find the time period of oscillation. AN INITIATIVE BY APT KOZHIKODE

Solution

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T = 2\pi\sqrt{\frac{2}{800}} = 2\pi\sqrt{\frac{1}{400}}$$
$$T = 2\pi \times \frac{1}{20} = \frac{\pi}{10} \text{ s}$$

Question (3): The displacement of a particle is given by $x = 4\sin(5t)$

Find:

- a) Amplitude
- b) Angular frequency
- c) Time period

Solution

Comparing with $x = A\sin(\omega t)$:

$$A = 4 \text{ m}$$
$$\omega = 5 \text{ rad/s}$$
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5} \text{ s}$$

Question (4): At what displacement is the kinetic energy equal to potential energy in SHM of amplitude A ?

Solution

Total energy: $E = \frac{1}{2}kA^2$

For equal KE and PE:

$$PE = \frac{E}{2}$$
$$\frac{1}{2}kx^2 = \frac{1}{4}kA^2$$
$$x^2 = \frac{A^2}{2} \Rightarrow x = \frac{A}{\sqrt{2}}$$

Question (5): The time period of a simple pendulum is 2 s. Find its length. ($g = 9.8 \text{ m s}^{-2}$)

Solution

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$2 = 2\pi \sqrt{\frac{l}{9.8}}$$

$$\sqrt{\frac{l}{9.8}} = \frac{1}{\pi}$$

$$l = \frac{9.8}{\pi^2} \approx 0.99 \text{ m}$$

Question (6): A particle has velocity 0.6 m s^{-1} at displacement 3 cm . Amplitude is 5 cm . Find angular frequency.

Solution

Velocity in SHM:

$$v = \omega \sqrt{A^2 - x^2}$$

$$0.6 = \omega \sqrt{(0.05)^2 - (0.03)^2}$$

$$0.6 = \omega \sqrt{0.0016}$$

$$\omega = \frac{0.6}{0.04} = 15 \text{ rad s}^{-1}$$

Question (7): A spring is stretched by 4 cm by a force of 8 N . Find the time period if a mass of 1 kg is attached.

Solution

$$k = \frac{F}{x} = \frac{8}{0.04} = 200 \text{ N m}^{-1}$$

$$T = 2\pi \sqrt{\frac{1}{200}} \approx 0.44 \text{ s}$$

Question (8): The acceleration of a particle in SHM is $-16x$. Find its time period.

Solution

$$a = -\omega^2 x \Rightarrow \omega^2 = 16 \Rightarrow \omega = 4$$

$$T = \frac{2\pi}{\omega} = \frac{\pi}{2} \text{ s}$$

Question (9): If the amplitude of SHM is doubled, how does total energy change?

Solution

$$E \propto A^2$$

If $A \rightarrow 2A$,

$$E \rightarrow 4E$$

Question (10): A block attached to a spring performs SHM. If the kinetic energy at mean position is 16 J, find total energy.

Solution

At mean position, PE = 0

So: Total Energy $E = KE_{max} = 16 \text{ J}$

Question (11): A simple pendulum oscillates with time period 2.4 s. Find the percentage change in time period if its length is increased by 4%.

Solution

$$\begin{aligned} T &\propto \sqrt{l} \\ \frac{\Delta T}{T} &= \frac{1}{2} \frac{\Delta l}{l} \\ &= \frac{1}{2} \times 4\% = 2\% \end{aligned}$$

Question (12): Find the ratio of velocities at displacement $A/2$ and $A/\sqrt{2}$. Where A is the amplitude.

Solution

$$v = \omega \sqrt{A^2 - x^2}$$

At $x = A/2$:

$$v_1 = \omega A \sqrt{1 - \frac{1}{4}} = \omega A \frac{\sqrt{3}}{2}$$

At $x = A/\sqrt{2}$:

$$\begin{aligned} v_2 &= \omega A \sqrt{1 - \frac{1}{2}} = \omega A \frac{1}{\sqrt{2}} \\ \frac{v_1}{v_2} &= \sqrt{\frac{3}{2}} \end{aligned}$$

Question (13): A particle executes SHM with maximum acceleration 4 m s^{-2} and maximum velocity 2 m s^{-1} . Find amplitude.

Solution

$$a_{max} = \omega^2 A \quad v_{max} = \omega A$$

$$\omega = \frac{a_{max}}{v_{max}} = \frac{4}{2} = 2$$

$$A = \frac{v_{max}}{\omega} = 1 \text{ m}$$

Question (14): Two SHMs of equal amplitude and frequency are superposed with phase difference π .

Find resultant amplitude.

Solution

$$\begin{aligned} A_{res} &= 2A \cos \frac{\phi}{2} \\ &= 2A \cos \frac{\pi}{2} = 0 \end{aligned}$$

Result: No oscillation

Question (15): The total energy of a particle in SHM is 10 J. Find its kinetic energy at displacement $A/2$.

Solution

$$PE = \frac{1}{2} k \left(\frac{A}{2} \right)^2 = \frac{1}{4} E$$

$$KE = E - PE = 10 - 2.5 = 7.5 \text{ J}$$

Question (16): A particle executes SHM with total energy E .

At what fraction of amplitude does the kinetic energy become three times the potential energy?

Solution

Total energy:

$$E = KE + PE$$

Given: $KE = 3PE$

So: $E = 4PE \Rightarrow PE = \frac{E}{4}$

In SHM: $PE = \frac{1}{2} kx^2, E = \frac{1}{2} kA^2$

$$\frac{x^2}{A^2} = \frac{1}{4}$$

$$\boxed{x = \frac{A}{2}}$$

Question (17): A particle performs SHM of amplitude A . Find the time taken to move from $x = A/2$ to $x = A/\sqrt{2}$.

Solution

Displacement equation: $x = A \sin(\omega t)$

At $x_1 = A/2$:

$$\sin(\omega t_1) = \frac{1}{2} \Rightarrow \omega t_1 = \frac{\pi}{6}$$

At $x_2 = A/\sqrt{2}$:

$$\sin(\omega t_2) = \frac{1}{\sqrt{2}} \Rightarrow \omega t_2 = \frac{\pi}{4}$$

Time difference:

$$\Delta t = \frac{1}{\omega} \left(\frac{\pi}{4} - \frac{\pi}{6} \right) \quad \boxed{\Delta t = \frac{\pi}{12\omega}}$$

Question (18): In SHM, find the ratio of maximum restoring force to maximum kinetic energy.

Solution

Maximum restoring force:

$$F_{max} = kA$$

Maximum kinetic energy:

$$KE_{max} = \frac{1}{2} kA^2$$

$$\frac{F_{max}}{KE_{max}} = \frac{kA}{\frac{1}{2}kA^2} \quad \boxed{\frac{2}{A}}$$

Question (19): At what displacement does acceleration equal velocity in magnitude?

Solution

$$v = \omega \sqrt{A^2 - x^2}, \quad a = \omega^2 x$$

Equating:

$$\begin{aligned} \omega^2 x &= \omega \sqrt{A^2 - x^2} \\ \omega x &= \sqrt{A^2 - x^2} \end{aligned}$$

$$\omega^2 x^2 + x^2 = A^2$$

$$x = \frac{A}{\sqrt{1 + \omega^2}}$$

Question (20): If the total energy of SHM increases by 21%, find percentage increase in amplitude.

Solution

$$E \propto A^2$$

$$\frac{\Delta E}{E} = 0.21 \Rightarrow \frac{\Delta A}{A} = \frac{1}{2}(0.21)$$

$$\boxed{10.5\%}$$

14. WAVES

Question (1): A transverse harmonic wave on a string is described by $y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4)$ where, x and y are in cm and t in second. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave?
If it is travelling, what are the speed and direction of its propagation?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the origin?

(d) What is the least distance between two successive crests in the wave?

Solution

a) Given equation

$$y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4)$$

Comparing this with standard form

$$y(x, t) = a \sin(\omega t + kx + \phi)$$

Hence, the given equation represents travelling wave from right to left

We have

$$\omega = 36 \text{ rad/s}, k = 0.018 \text{ cm}^{-1}$$

$$\text{Now, frequency } \nu = \frac{\omega}{2\pi} \text{ and wavelength } \lambda = \frac{2\pi}{k}$$

Speed of wave

$$V = \frac{\omega}{k} = \frac{36}{0.018} = 2000 \text{ cm/s} = 20 \text{ m/s}$$

b) Amplitude, $a = 3.0 \text{ cm}$

frequency of the given wave

$$\nu = \frac{\omega}{2\pi} = \frac{36}{2 \times 3.14} = 5.73 \text{ Hz}$$

c) Phase angle $\Phi = \frac{\pi}{4}$

d) The least distance between two successive crest or trough is equal to the wavelength λ of the wave

As
$$k = \frac{2\pi}{\lambda}$$
$$\lambda = \frac{2\pi}{k}$$
$$= \frac{2 \times 3.14}{0.018} = 348.88 \text{ cm} = 3.49 \text{ m}$$

Question (2): A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at $20^\circ\text{C} = 343 \text{ m/s}$?

Solution

Given, velocity of the wave $V = 343 \text{ m/s}$

Required tension $T = ?$

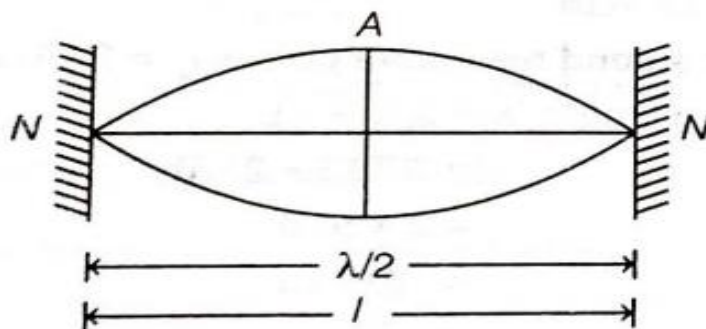
Mass density $\mu = \frac{m}{l} = \frac{2.10}{12} = 0.175 \text{ kg/m}$

Speed of transverse wave in wire

$$V = \sqrt{\frac{T}{\mu}}$$
$$T = v^2 \mu = (343)^2 \times 0.175$$
$$= 20588.575 \text{ N}$$
$$\approx 2.06 \times 10^4 \text{ N}$$

Question (3): A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of wire is $3.5 \times 10^{-2} \text{ kg}$ and its linear mass density is $40 \times 10^{-2} \text{ kg/m}$. What is (a) the speed of a transverse wave on the string and (b) the tension in the string?

Solution



In fundamental mode of vibration

$$\frac{\lambda}{2} = l \text{ or } \lambda = 2l$$

As

$$v = 45 \text{ Hz}$$

$$\mu = \frac{m}{l} = 4 \times 10^{-2} \text{ kg/m}$$

$$m = 3.5 \times 10^{-2} \text{ kg}$$

$$l = \frac{m}{\mu} = \frac{3.5 \times 10^{-2} \text{ kg}}{4 \times 10^{-2}} = 0.845 \text{ m}$$

Now from Eq. (i)

a) $V = v\lambda = v2l$

$$V = 45 \times 2 \times 0.875 = 78.75 \text{ m/s}$$

b) as $V = \sqrt{\frac{T}{\mu}}$

$$T = V^2(\mu) = (78.75)^2 \times 4.0 \times 10^{-2} = 248.06 \text{ N}$$

Question (4): A string of mass 2.50 kg is under a tension of 200 N. The Length of the stretched string is 20.0 m. If the transverse jerk is Struck at one end of the string, how long does the disturbance take to reach the other end?

Solution

Velocity of Transverse Wave

$$V = \sqrt{\frac{T}{\mu}}$$

$T = \text{Tension in the string} = 200 \text{ N}$

$\mu = \text{Mass per unit Length}$

$$= \frac{\text{Mass}}{\text{Length}} = \frac{2.50}{20} = 0.125 \text{ kg/m}$$

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40 \text{ m/s}$$

Time Taken by the disturbance to reach the other end

$$t = \frac{l}{V} = \frac{20}{40} = 0.50 \text{ s}$$

Question (5): A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340 m/s and in water 1486 m/s..

Solution

a) The reflected sound travels in the air, so

Speed of sound in air $V = 340 \text{ m/s}$, Frequency $\nu = 1000 \times 10^3 \text{ Hz}$

$$\text{Wavelength of sound } \lambda = \frac{V}{\nu} = \frac{340}{10^6} = 3.4 \times 10^{-4} \text{ m}$$

b) Speed of sound wave in water, $V_w = 1486 \text{ m/s}$

$$\begin{aligned} \text{Wavelength of transmitted wave} = \lambda_t &= \frac{V_w}{\nu} = \frac{1486}{10^6} \\ &= 1.49 \times 10^{-3} \text{ m} \end{aligned}$$

Question (6): A hospital uses an ultrasonic scanner to locate tumour's in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 km/s? The operating frequency of the scanner is 4.2 MHz.

Solution

Speed of wave $v = 1.7 \text{ km/s} = 1.7 \times 10^3 \text{ m/s}$

Operating frequency of the scanner $\nu = 4.2 \text{ MHz}$
 $= 4.2 \times 10^6 \text{ Hz}$

Wavelength of sound in the tissue $\lambda = \frac{v}{\nu} = \frac{1.7 \times 10^3}{4.2 \times 10^6}$
 $\approx 4.1 \times 10^{-4} \text{ m}$

Question (7): A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower.

When is the splash heard at the top given that the speed of sound in air is 340 m/s? ($g = 98 \text{ m/s}^2$)

Solution

Height of the tower (h) = 300m

Initial Velocity $u = 0$

Acceleration due to gravity = 9.8 m/s^2

Speed of Sound in air = 340m/s

Time taken by stone to reach the pond = t_1

Using the equation of motion $h = ut + \frac{1}{2}gt_1^2$

$$300 = 0 + \frac{1}{2} \cdot 9.8 \cdot t_1^2$$

$$t_1 = \sqrt{\frac{300 \times 2}{9.8}} = 7.82 \text{ s}$$

Time Taken by the sound to reach the top of the tower

$$t_2 = \frac{h}{v} = \frac{300}{340} = 0.88 \text{ s}$$

$$\text{Total Time } t = t_1 + t_2 \\ = 7.82 + 0.88 = 8.7 \text{ s}$$

Question (8): A wire of length l , and mass per unit length $6 \times 10^{-3} \text{ kg/m}$ is put under a tension of 540 N.

Two consecutive frequencies that it resonates at are 420 Hz and 490 Hz. Find length l .

Solution

For a string fixed at both ends,

$f_n = n \times \text{fundamental frequency}$

Given $f_n = 420 \text{ Hz}$ and $f_{n+1} = 490 \text{ Hz}$ are consecutive,

$f_{n+1} - f_n = 490 - 420 = 70 \text{ Hz} = \text{fundamental frequency}$

Fundamental frequency = $v/2L = 70 \text{ Hz}$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{540}{6 \times 10^{-3}}} = 300 \text{ m/s}$$

$$70 = 300 / (2L)$$

$$L = 300 / (2 \times 70) = 2.14 \text{ m}$$

So, length $l \approx 2.14$ m

Question (9): In a closed organ pipe of length 105 cm, standing waves are set up corresponding to third overtone. What distance from the closed end, a pressure node is formed?

Solution

For a closed organ pipe, the third overtone corresponds to the 7th harmonic ($n = 7$, where $n = 1, 3, 5, \dots$).

Pressure nodes occur at:

$$x = (2k + 1)\lambda/4, \text{ where } k = 0, 1, 2, \dots$$

For the 7th harmonic:

$$\lambda = 4L/7 = 4 \times 105/7 \text{ cm} = 60 \text{ cm}$$

Pressure nodes are at:

$$\begin{aligned} x &= \lambda/4, 3\lambda/4, 5\lambda/4 \\ &= 15 \text{ cm}, 45 \text{ cm}, 75 \text{ cm} \end{aligned}$$

Question (10): Two sound waves are represented by $y_1 = 4 \sin 200\pi t$ and $y_2 = 3 \sin 208\pi t$ superimpose each other. Find the beat frequency.

Solution

$$\omega_1 = 200\pi \Rightarrow f_1 = 100 \text{ Hz}$$

$$\omega_2 = 208\pi \Rightarrow f_2 = 104 \text{ Hz}$$

$$\text{Beat frequency} = f_2 - f_1 = 104 - 100 = 4 \text{ Hz}$$

Question (11): A plane progressive wave is represented by the equation $y = 5 \sin(8\pi t - 0.2x)$ meter. Find the velocity of the wave.

Given wave equation:

$$y = 5 \sin(8\pi t - .2x) \text{ meter.}$$

Solution

Comparing with standard form:

$$y = A \sin(\omega t - kx)$$

$$\omega = 8\pi \text{ rad/s}$$

$$k = 0.2 \text{ rad/m}$$

$$\text{Velocity (v)} = \omega/k$$

$$= 8\pi / 0.2$$

$$= 40\pi \text{ m/s}$$

Question (12): If the length of a stretched string is shortened by 20% and the tension is increased by 21%. Find the percentage change in frequency.

Solution

Given:

$$L_2 = 0.8 L_1$$

$$T_2 = 1.21 T_1$$

$$f_1 = (1/2L_1) \times \sqrt{\frac{T_1}{\mu}}$$

$$f_2 = (1/2L_2) \times \sqrt{\frac{T_2}{\mu}}$$

$$\begin{aligned} \frac{f_2}{f_1} &= \left(\frac{L_1}{L_2}\right) \times \sqrt{\frac{T_2}{T_1}} \\ &= \left(\frac{1}{0.8}\right) \times \sqrt{1.21} \\ &= 1.25 \times 1.1 \\ &= 1.375 \end{aligned}$$

$$\begin{aligned} \text{Percentage change in frequency} &= ((f_2 - f_1)/f_1) \times 100\% \\ &= (1.375 - 1) \times 100\% \\ &= 37.5\% \end{aligned}$$

So, the frequency increases by 37.5%

Question (13): A 1.5 m long rope is stretched between two supports with a tension that makes the speed of transverse wave 48 m/s. Find the wavelength of second overtone.

Solution

For a string fixed at both ends, the wavelength of the n th harmonic is given by $\lambda = 2L/n$.

Second overtone means $n = 3$ (fundamental = 1, first overtone = 2, second overtone = 3).

Given $L = 1.5$ m, $n = 3$

$$\lambda = 2L/n = (2 \times 1.5)/3 = 1 \text{ m}$$

Question (14): A string 2m long and fixed at its two ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. Find the speed of the wave and its fundamental frequency.

Solution

Given:

$$L = 2 \text{ m}$$

$n = 3$ (third harmonic)

$$f_3 = 240 \text{ Hz} = 3f_1$$

$$f_3 = 3\left(\frac{v}{2l}\right)$$

$$240 = 3\left(\frac{v}{2 \times 2}\right)$$

$$v = (240 \times \frac{2 \times 2}{3}) = 320 \text{ m/s}$$

$$\text{Fundamental frequency } f_1 = \frac{v}{2l} = \frac{320}{2 \times 2} = 80 \text{ Hz}$$

Question (15): The transverse displacement of a string (clamped at its both ends) is given by

$$y(x,t) = 0.06 \sin \frac{2}{3} x \cos (120\pi t)$$

where, x and y are in metre and t in second. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg.

Answer the following

(a) Does the function represent a travelling wave or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency and speed of each wave?

(c) Determine the tension in the string.

Solution

a) The given equation $y(x,t) = 0.06 \sin \left(\frac{2\pi}{3} x\right) \cos (120\pi t)$ is of the form

$$y(x,t) = 2a \sin kx \cos \omega t$$

hence it represents a stationary wave

b) The two waves producing the standing wave

$$y(x,t) = 2a \sin kx \cos \omega t$$

are due to superposition of these waves

$$y_1(x,t) = a \sin (\omega t - kx)$$

$$y_2(x,t) = a \sin (\omega t + kx)$$

We have

$$y(x,t) = 0.06 \sin \left(\frac{2\pi}{3} x\right) \cos (120 \pi t)$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3} \quad \lambda = 3 \text{ m}$$

also, $\omega = 120\pi \text{ rad/s}$

angular frequency $\omega = 2\pi v \rightarrow v = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi}$

$$v = 60\text{Hz}$$

Wave Speed $v = v\lambda$
 $= 60 \times 3$
 $= 180 \text{ m/s}$

and $y_1 = 0.03 \sin(120\pi t - \frac{2\pi}{3}x)$

$$y_2 = 0.03 \sin(120\pi t + \frac{2\pi}{3}x)$$

c) Speed of wave $v = \sqrt{\frac{T}{\mu}}$

Mass per unit length $\mu = \text{Mass/length} = \frac{3.0}{1.5} \times 10^{-2}$

$$= 2 \times 10^{-2} \text{ kg/m}$$

$$T = \mu v^2$$

$$T = 0.02 (180\pi)^2$$

$$= 6.4 \times 10^3 \text{ N}$$

1.. Units and measurement... Dayana

2.. Motion in a straight line.... Shyni C

3 Motion in a plane... Jithesh k k

4.Laws of motion.... Sabitha

5.Work energy power... Shameer

6 Rotational motion Sabitha E

7 Gravitation.... Sabira

8 properties of solids.... Irshad

9 Properties of fluids Sitara

10 Thermal properties... Prajeesh

11 Thermodynamics... Riju

12 Kinetic theory.. Dayana

13 Oscillations. Sijuraj

14 Waves...Archana.N

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